

Benjamin H. Yandell,

*The Honors Class: Hilbert's Problems and Their Solvers*

Natick, Massachusetts: A K Peters, 2002

ix + 486pp. ISBN 1568811411

## REVIEW

WIM RUITENBURG

By the end of the nineteenth century it became more common for eminent scholars to turn to programmatic or foundational questions. Among mathematicians, David Hilbert was a prime example. His *Zahlbericht* of 1897, a report on the status of algebraic number theory, enriched with many original observations, was an organization of this entire branch of mathematics. His *Grundlagen der Geometrie* of 1899 was the first precise axiomatic study of geometry, significantly refining the foundational geometry done since Euclid. It was only natural for Hilbert to use his lecture at the 1900 (second) International Congress of Mathematicians in Paris to attempt to lift the veil behind which the future of mathematics lay hidden. Different from Henri Poincaré's presentation at the 1897 ICM in Zürich, Hilbert chose to execute his intentions through a list of Problems. By so doing he offered current and future mathematicians immediate questions to work on. Ten Problems were briefly discussed during the lecture. The complete list of 23 was made available in print.

The success of the list may have even surprised Hilbert. The individual Problems widely vary in significance, difficulty, and clarity. The third Problem was solved before its official publication. Others are still open. Some Problems are very specific, while others are research programs. One is wrong, or at least needs serious re-statement. The solutions to some Problems, particularly Problems 10 and 13, are contrary to Hilbert's expectations.

Yandell's book joins a vast literature on Hilbert's Problems and related matters. There are many ways to tell the story of its effects on mathematics and the mathematical community. Since no single book can cover everything, we consider two questions. What aspects stand out in the book under review? How well are these aspects presented?

A seriously mathematical approach is in [1, 2]; Yandell's book is intended for a broad audience. We encounter only a sketched mathematical content, intended to give the general reader a sense of what it is about. Another current and well-known book on the Hilbert Problems is Jeremy Gray's [3]. Yandell's book has less than 500 pages; Gray's is even shorter. Fortunately, the two authors chose different selection criteria to limit the sizes of their books. We briefly discuss Gray's book to allow us to emphasize what is special about Yandell's. Gray chose a mixed historical-mathematical approach, following the events essentially in historical order. After introductory chapters on Hilbert before 1900, and on Poincaré, the responses are divided into historical periods; up to and including the first world war; the inter-war years; and the postwar period. Although Gray avoids profound mathematical arguments, some academic preparation is expected from his readers, which should include the readership of *The Review of Modern Logic*. Historical material focuses on mathematical events, or events that involve collegiate, political, or national and international events that clearly interfere with mathematical activity of significance to the story. What choices did Yandell make?

Superficially, Yandell writes about the same content. In reality, beyond the obligatory details, there are significant differences, both in style and content. Their bibliographies are largely disjoint. Yandell does not follow the historical order of events. The table of contents shows the alternate choice:

INTRODUCTION: THE ORIGIN OF THE COORDINATES (21 pages)

**Introduction**

**Advice: How to Read This Book**

**The Origin of the Coordinates**

THE FOUNDATION PROBLEMS { 1, 2, 10 } (91 pages)

**Set Theory, Anyone?**

**I Am Lying (Mathematics Is Consistent)**

**The Perfect Spy: How Many Real Numbers Are There?**

**Can't We Do This with a Computer?**

THE FOUNDATIONS OF SPECIFIC AREAS { 3, 4, 5, 6 } (47 pages)

**In the Original**

**Distance**

**Something for Nothing**

**On Again, Off Again: Physics and Math**

NUMBER THEORY { 7, 8, 9, 11, 12 } (95 pages)

**First, State the Tune**  
**Transcending Local Conditions**  
**The Inordinate Allure of the Prime Numbers**  
**Castles of Air**

ALGEBRA AND GEOMETRY: A MISCELLANY { 14, 15, 16, 17, 18 }  
 (33 pages)

**What Is Algebra?**  
**Schubert's Variety Show**  
**Graph That Curve**  
**How Many Kinds of Crystals Are There, and**  
**Does the Grocer Know How to Stack Oranges?**

THE ANALYSIS PROBLEMS { 13, 19, 20, 21, 22, 23 } (89 pages)

**Analysis Takes at Least Seven Years**  
**How Famous Can a Function Theorist Be?**  
**Schools Amid Turbulence**  
**Past Chernaya Rechka, to 61 Savushkina Street**  
**Work on It**

WE COME TO OUR CENSUS (1 page)

**Census**

There is an APPENDIX, with the 1902 Bulletin of the AMS version of the 23 Problems, with footnotes, a selected bibliography, and a limited index.

The first SECTION of the book sketches Hilbert's life. The other SECTIONS partition the Problems by category. They also stay close to the order of the Problems as laid out a century before. The correlation between classification and order implies that this reflects Hilbert's design. Only two Hilbert Problems moved: Problem 10 on the solvability of all Diophantine equations, moved up from number theory to foundations because of its negative solution. Problem 13 on the reach of compositions of two-variable functions, moved down from algebra to analysis, because its positive solution, contrary to Hilbert's negative expectation, uses techniques familiar to analysts. If Gray's layout runs the risk of forcing his narrative to jump back and forth between different Hilbert Problems while trying to stay within the same time period, Yandell's runs the risk of jumping back and forth in time, when moving from SECTION to SECTION. For example, Kolmogorov plays a significant role in three different SECTIONS. There are two reasons why this is only a minor problem. First, within individual SECTIONS,

Yandell re-orders Hilbert Problems when it serves his purposes. Second, the range of mathematics vastly expanded during the twentieth century. Consequently, most mathematicians got seriously involved in only one area contained in a single SECTION. One level closer, we get to the **Subsections**. The obscurity of many of its titles stands in clear contrast to all but one of the SECTION titles. In what sense do we get **Something for Nothing**? Why do we want to go **Past Chernaya Rechka, to 61 Savushkina Street**? While the meaning of their titles becomes clear after reading the **Subsections**, the choice sometimes doesn't.

The book is intended for a broad audience, so there is little serious mathematics. The best mathematical parts consist of well-known and nice illustrations of mathematical ideas and concepts. These include Cantor's diagonal argument (page 32), measure zero set (page 172), and transcendental number (pages 172–173). If a part requires some mathematical sophistication, then the reader may choose to skip it and can follow the non-mathematical part without difficulty. The story is about mathematicians and their connections, rather than about mathematics.

Most of the principal figures in the stories are members of the Honors Class, solvers of one or more of the Hilbert Problems. Naturally, its members do not receive equal representation. I can imagine three legitimate reasons for the variation. First, Yandell must be able to write a story. In some cases the quest was more fruitful than in others. Second, the story must be sufficiently interesting to a broad audience. Third, Hilbert's Problems are not all of equal importance. The list of those who receive significant coverage includes Paul Cohen in the Foundations SECTION; Max Dehn in the third SECTION; Carl Ludwig Siegel and Emil Artin in the fourth SECTION on number theory; Ludwig Bieberbach in the fifth; and Henri Poincaré and Andrei Nikolaevich Kolmogorov in the sixth SECTION on analysis. Of course, there is the sketch of Hilbert's life in the first SECTION. Besides discussion of members of the Honors Class, there are significant stories about other scholars. The mathematical community tends to give all the recognition for solving a problem to the one who completes the final "major" step in the proof. It is not always clear what constitutes the final "major" step. Recipients of recognition are sometimes embarrassed by how much they receive beyond their own sense of what is fair to attribute to themselves. Yandell sidesteps most of these issues by simply including significant contributors of earlier partial results. For example Gleason, Zippin, and Montgomery are included on Problem 5. Other scholars appear to be included for no serious reason other than for providing

good story material, like Ramanujan (pages 209–211). These individual stories also vary greatly, probably for reasons similar to the above about the members of the Honors Class.

The personal stories form the outstanding aspect of Yandell's book. Though entertaining, their historical significance is debatable. We learn a lot of personal detail. In the first SECTION we read that Hilbert was a good dancer, and a flirt. Both he and his wife Käthe Jerosch helped make Göttingen a more welcoming environment for scholars in general, and for students in particular. On page 10 there is a translation of a brief poem about Göttingen that Hilbert scribbled in one of his notebooks. The text on Cantor ends with parts of a poem by him. Gödel's mother was a gymnast and a good ice skater (page 42). We read a lot of detail on Black Mountain College in North Carolina, where Max Dehn spent his later years (pages 131ff). And so on. Yandell's sources include well-known books like Constance Reid's biography of Hilbert, and Hel Braun's autobiographical work. The most original parts derive from his personal communications with members of the Honors Class, or with people who were very close to them.

We can hardly get more personal than by discussing a human being in terms of social relations of a biological nature. We already know that Hilbert was a good dancer, and a flirt. On page 66, we read that van Heijenoort "epitomized cool and was attractive to women, and this continued *even* after he became a logician" (emphasis added). He had lots of relations, including an affair with Frida Kahlo. Finally it was a woman who killed him. It is suggested on page 77, that Paul Cohen regained his enthusiasm for solving Hilbert 1 by driving through one of the most beautiful landscapes in the world with a beautiful woman by his side. Collaborators Zippin and Montgomery appeared to make a mismatched pair (page 157): Montgomery was tall, Midwestern, a prototype WASP, obviously handsome; Zippin was a short New Yorker, culturally Jewish. When, in response to Erdős pointing out a nice "epsilon" (referring to a little child), Ulam points out a capital epsilon (its beautiful mother), Erdős blushed with embarrassment (page 166). We could go on like this. Relevant to the history of mathematics? Maybe. More likely it contributes to Yandell's attempt to provide a "connect-the-dots" for part of twentieth century mathematical culture (page 4).

At times the flow of the text feels 'choppy,' as if small parts were taken from different index cards, and simply put together. Sometimes these tidbits are interesting, sometimes mystifying. For example, in the nice **Subsection** on Hilbert 10, we read that "I did what I could" characterizes Julia Robinson's whole career (page 100). The opinions

of an author can play a natural role in the book he writes. However, Yandell often states his opinions without further supporting evidence. Examples: “Cantor believed that positivism and materialism had done great damage to religious culture, that Newton’s *Principia* had played a central role in this (as I am sure it has)” (page 34). “The real content of [Hilbert’s] philosophy, as I see it, was that he wanted to protect the greatest possible range of mathematical discourse from criticism” (page 40). “Cohen used this approach in his book because he thought it conveys the key ideas more clearly (and I agree), ...” (page 78). In relation to Hardy’s opinions and the usefulness of number theory, “[t]here are reasonable people who would disagree, but I think they are wrong” (page 167). “Hales’s computer-assisted proof is either right or wrong. Time and work will get the situation sorted out, and I suspect that Hales’s proof will stand” (page 290). On Darboux’s obituary of Poincaré, “[h]is account has a tone of Olympian praise, but I believe it was written with care and is accurate” (page 300). Yandell’s opinions appear to be sensible but are poorly supported, if supported at all. His view of Hilbert’s philosophy appears more appropriate than Arnold’s (page 41), but it is not new.

Ideally, Yandell’s stories present a properly balanced overview from which we can fairly “connect the dots” of mathematical culture for about the first half of the twentieth century. Unfortunately, the *Annalen* Affair is essentially missed. The Affair is well-published. After Hilbert returned from the 1928 ICM in Bologna, he summarily ‘fired’ Brouwer as editor of the *Mathematische Annalen*. Did Hilbert have the right to do so? Brouwer had been a conscientious, competent, active editor. Then what were Hilbert’s reasons for trying to sack Brouwer? Yandell’s only serious reference to these events is the single sentence “After the conference, Hilbert succeeded in having both Brouwer and Bieberbach removed from the editorial board of the *Annalen*” (page 285). To get some idea of how much is left out, see Smoryński’s article [4]. Even for someone who questions details of Smoryński’s version, Yandell’s sentence is misleading. One may explain Hilbert’s action more convincingly as an impulsive act of a person of great authority, who ‘lost patience’ with someone he could not sufficiently control. Hilbert’s instruction was implemented and rationalized after the event, as reflected in Jeremy Gray’s claim “Hilbert decided that he had to protect the *Mathematische Annalen*, ...” (see [3, page 167]). Significant scholars outside Hilbert’s orbit of power did not support the re-organization of the editorial board of the *Annalen*. Yandell’s one-sentence comment on page 285 would be more accurate if rewritten as

“After the conference, Hilbert succeeded in having Brouwer, Bieberbach, and Einstein removed from the editorial board of the *Annalen*.”

Yandell gives us a glimpse of the mathematical culture of part of the twentieth century. He has uncovered details that would have been lost were it not for his personal efforts. Unfortunately, Yandell appears to have chosen to omit elements crucial to providing a balanced and fair “connect-the-dots” for the mathematical culture of the period.

#### REFERENCES

- [1] Felix E. Browder (editor), *Mathematical Developments Arising from Hilbert Problems. Proceedings of Symposia in Pure Mathematics*, Vol. XXVIII, Part 1, Providence, RI: American Mathematical Society, 1976.
- [2] Felix E. Browder (editor), *Mathematical Developments Arising from Hilbert Problems. Proceedings of Symposia in Pure Mathematics*, Vol. XXVIII, Part 2, Providence, RI: American Mathematical Society, 1976.
- [3] Jeremy J. Gray, *The Hilbert Challenge*, Oxford: Oxford University Press, 2000.
- [4] Craig Smoryński, “Hilbert’s programme”, *CWI Quarterly* **1**, No. 4, 1988, 3–59.

DEPARTMENT OF MATHEMATICS, MARQUETTE UNIVERSITY, P.O. BOX 1881,  
MILWAUKEE, WI 53201

*E-mail address:* `wimr@mscs.mu.edu`