Brouwer's Intuitionism

W. P. van Stigt

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A review by Wim Ruitenburg.

Luitzen Egbertus Jan Brouwer was not a nice man. His egotistic and often immature behaviour ultimately turned his closest colleagues, students, and friends away from him. The great appreciation of and interest in Brouwer that many mathematicians and philosophers have are for his genius. His peculiar self-centeredness and disregard for other people's opinions created one of the most original and profound philosophers of mathematics.

Walter van Stigt has gone to great lengths to provide us with an in-depth view of Brouwer, his philosophy, and his foundations of mathematics. He presents the whole of Brouwer's systematic speculation on the origin and nature of mathematics, requiring the inclusion of Brouwer as a person, but using Brouwer's writings as his main source of information. Of these there are more available now than when Heyting and Freudenthal prepared Brouwer's Collected Works, such as newly discovered manuscripts of unfinished books [2]. The need to study Brouwer as a person is made overwhelmingly clear in the pages of this book. It is no coincidence that it playfully consists of six chapters, for the magic number six reappears as the number of chapters in each of Brouwer's three unfinished books. Even Brouwer's dissertation, On the Foundations of Mathematics of 1907, was originally supposed to have six chapters, but was compressed into three when financial support finally ran out and time became of the essence. In the delimited scope of the book under review there are essentially no discussions of the contributions of others to or mention of later developments of intuitionism. An important exception is a relevant discussion of the formalization of intuitionism by Heyting and others. A critical comparison with other philosophies is, however, absent. Brouwer's contributions to topology are discussed only in the context of its contribution to his receiving a professorship at Amsterdam University and to his increased status in the international mathematical community. There is evidence that Brouwer chose the field of topology for just these purposes, a choice which seems natural for Brouwer, who preferred the geometric 'visual' approach, in line with his 'seeing with the inner eye'. In his public address The Nature of Geometry Brouwer equated 'topological' with 'geometrical' and 'formula-less'.

Chapter 1 of van Stigt's book contains a complete bibliography, including relevant unpublished papers and newspaper articles. Chapter 2 places Brouwer's work in its historical context,

with biographical details as are relevant and helpful for an understanding of the man Brouwer and the development of his ideas. Chapter 3 focuses on those of Brouwer's main philosophical ideas that bear directly on his interpretation of mathematics, such as the nature of man and mind, human knowledge, causality and the 'transcendental values of goodness, truth, and beauty'. Chapter 4 analyses the Brouwerian conception of mathematics, its genesis in the Primordial Intuition, its characteristic constructions and its essential 'Subjectivity'. Chapter 5 starts from Brouwer's interpretation of the origin of language and follows his development of a theory of semantics and his critique of the traditional uses of logical principles and of formalization in mathematics. Chapter 6 introduces some of the fundamental parts of Brouwer's intuitionist reconstruction of mathematics, particularly those which deviate from the classical treatment and reveal their philosophical origins. The appendices represent a selection of Brouwer's less commonly accessible writings.

Brouwer was born in 1881. We are told little about his earliest years except that he was two years ahead of his class and still excelling in all academic topics (except maybe for 'effort' in mathematics). When he entered Amsterdam University in 1897, his genius was soon recognized. Although welcomed by the student societies, Brouwer's mixture of shyness and arrogance kept him away from his fellow mathematicians. The first documented evidence of Brouwer's egotistic, confrontational nature is the 1898 presentation of his PROFESSION OF FAITH at his confirmation ceremony in church. His expression of a personal, rather than a collective or external, recognition of God and his evaluation of organized religion as right (only) for the stupid masses, show an unusual intellect as well as a lack of sensitivity towards the congregation present at the occasion. Neither here nor later does van Stigt spare Brouwer. Although Brouwer was sensitive and was very much guided by his feelings, he had "a growing need to be loved, admired and recognized; yet his dealings with others are dominated by a certain meanness, by personal ambition and suspicion of others. He was subject to moods of pessimism and depression and could fly into violent tempers. During such spells of emotional upset work remained impossible" (page 24).

Brouwer was close to abandoning mathematics and its role serving the sciences when Gerrit Mannoury, a self-taught mathematician and one of the first to introduce the fundamental questions of the time on the foundations of mathematics into the Netherlands, opened his eyes. Brouwer's manifesto LIFE, ART AND MYSTICISM of 1905, written in opposition to the philosopher Bolland, contains the first published references to Brouwer's philosophical views: the excellence of 'inner vision', or 'intuition', the Subject, and the place of language. But it was also a manifesto

of "an angry young man", fulminating against applied sciences (on medical sciences: "The medical industry was with barbers and quacks in good hands; practised within the confines of the intellect, as medical science, it is far less effective ..." (page 31)) and against social acting, more directly associated with women ("There is less difference between a woman in her innermost nature and an animal such as a lioness than between two twin brothers ..." (page 32)). In the end, the manifesto appears to be a mixture of profound insights and frustrations, combined with a desire to shock the 'establishment'. Brouwer endorsed LIFE, ART AND MYSTICISM all his life. In fact, with rare exceptions, Brouwer stood by all his previous publications, seldom being able to find any faults in them.

Van Stigt puts it mildly when he makes the observation that Brouwer's strong condemnations contain a touch of insincerity. Brouwer "ridicules fashions and human weaknesses which mark his own life, such as ambition, lust for power, jealousy ... His condemnation of those seeking security by amassing capital rings rather hollow in a man whose life was so obsessed with money; his sarcasm when he deals with spiritism, theosophy ... raise a smile in those who knew him ... (he attended seances and theosophic meetings ...)" (page 34). This trait is noted again and again. Brouwer uses his personal influence to get Mannoury a chair in mathematics and one of his brothers a chair in geology, while almost at the same time leading a public campaign for an open system of academic appointments on the sole ground of academic excellence and against the tradition of secrecy and personal preference (pages 68–69). Forgetting his moral disapproval of applied mathematics, Brouwer claims the setting-up of a national laboratory for photogrammetry to be his life's ambition (page 79). Brouwer's enthusiastic entry into public life in 1912 puts a question mark over the seriousness of his quest for solitude and the sincerity of his views on human society; and there is a complete reversal in his attitude toward social reform (page 194).

Brouwer's dissertation itself is essentially devoid of moralistic, 'mystical', and fanatic excursions. This happened partly under pressure from his 'promotor' Korteweg. A careful look at the correspondence between Korteweg and Brouwer and a global reading of the rejected parts of the thesis, both in van Stigt's book and in the original Dutch [1], show the sensitivity and pragmatic insight of Korteweg. Despite indications of some tension between student and professor, the correspondence shows essentially no argument against Korteweg's questioning of the need for including in the dissertation a pessimistic and mystical attitude of life. My reading of the Dutch original also showed the great care given to the English translations. Van Stigt makes it clear that Korteweg appreciated Brouwer's intellect and even gave up his own position at

the university to provide an Ordinary Professorship for Brouwer in 1913.

From his philosophy of mathematics Brouwer draws the remarkable conclusion that the Principle of the Excluded Middle of logic is not reliable. His paper THE UNRELIABILITY OF THE LOGICAL PRINCIPLES of 1908, written in Dutch, did not attract the attention it deserved. Van Stigt notes that even Brouwer himself originally did not appreciate its revolutionary character. Even in 1912 his attack on the Law of Excluded Middle is only an added footnote to the English translation of his inaugural address. Only in 1923 does Brouwer return to logic, criticizing the principle of double negation elimination $\neg\neg\varphi \to \varphi$. By November he discovers $\neg\neg\neg\varphi \leftrightarrow \neg\varphi$. The attacks on the Principle of the Excluded Middle became a propagandistic rallying point. The development of intuitionistic logic, however, is left to Kolmogorov, Glivenko, and, finally, Heyting in 1928, just when Brouwer retires into silence.

The end of Brouwer's period of intense activity arrives in a fashion in keeping with the general impression that van Stigt gives us of Brouwer as a difficult person. In this case, however, the blame doesn't fall on Brouwer. During the 1920's Hilbert rose to oppose Brouwer's views of mathematics and to support his own views on formalism. To what extent his opposition was to the ghost of Kronecker rather than to intuitionism, we don't know; a particularly lively description of the affair is given in [3], which is heartily recommended for additional reading. In 1928 the situation reached a climax when Hilbert headed a German delegation to the International Congress of Mathematicians at Bologna, and Brouwer supported a German boycott because of the way Germans had been discriminated against at these meetings since the end of World War I. After the Congress, Hilbert dismissed Brouwer as member of the editorial board of the MATHEMATISCHE ANNALEN without having the required approval of the other chief editorial members, Einstein and Carathéodory. Van Stigt's account shows how many more people were at fault while trying to protect Hilbert's reputation at the expense of Brouwer.

The ANNALEN Affair may not have been the only cause of Brouwer's withdrawal from the intuitionist programme. Brouwer's reconstruction had run aground, partly because of his difficulties in finding a satisfactory proof of his Fundamental Theorem (also called Bunch or Fan Theorem), partly also because, according to van Stigt, Brouwer's doctrine of the absolute separation of mathematics and language led to the 'unbearable awkwardness of his Intuitionist Mathematics'. Moreover, his plans for a Mathematical Institute in Amsterdam had gotten nowhere, while in the Fall of 1927 a new Mathematics Institute had opened in Göttingen, reflecting a shift in the centre of gravity of mathematical research.

Van Stigt spends close to 300 pages on a very detailed discus-

sion of Brouwer's philosophy in general, his philosophy of mathematics in particular, and on his ideas about language and logic (Van Stigt's sense for detail occasionally forced me to pay careful attention to some details of the Dutch language, e.g., to the distinction between 'willekeurig' and 'willekeurige' on page 179, between arbitrarily given and arbitrarily chosen). There is some overlap and repetition of ideas between these sections, but that is not fully avoidable without loss of clarity. Some repetition seems to be unnecessary, for example the many repeated explanations of the meaning of the Primordial Intuition.

This sturdily built hardbound book is nicely turned out, with nice fonts and with some black-and-white photographs. There are, unfortunately, an unnecessarily large number of misprints. Most of them ('philosphy' or repeated words [the the, of of]) are easily recognized. It creates a suitable, albeit unfortunate, feeling of uncertainty about some of the texts (Besides the missing quotation mark, is the 'l' in 'world' in error on page 200 in the quote 'The world cannot wait for 'higher society' while the 'higher society waits for the word'?).

Because of the limited scope of this book, there is no critical discussion of most of Brouwer's philosophy, leaving several questions begging for an answer. For example, Brouwer may arguably claim that current languages may not enable us to express all our thoughts and thought processes in word or on paper, but there is no reason to believe that human memory is more reliable than 'written' memory. The distinction between the two is further blurred for those who choose to doubt the existence of an external world and may see writing on paper as a trick of the human mind to remember information. Brouwer approaches this problem when he admits the fallibility and limitations of human memory and expresses appreciation for language as an aid to memory. Brouwer also once claimed that for pure mathematics there cannot be an infallible language which excludes misunderstanding in communication. Such a statement, however, seems to be metalinguistic by making a claim about all possible languages based on experience with only a few. The most that should be claimed is that we can't recognize an infallible language, possibly even when we have one. So language is only *less* certain. But even this weakened claim of uncertainty makes it too easy to explain away problems in logic 'because language is unreliable anyway'. Brouwer's problems with the Fundamental Theorem (pages 93, 383) may be seen in the light of his not carefully maintaining distinctions between higher order levels. Some of Brouwer's later uses of metamathematical and higher-order observations lack the care given to and rigor taken with early intuitionistic mathematics itself. Particularly debatable results are Brouwer's new principle of Bar Induction, described as 'evident from profound intuitionist

reflection', and his use of an analysis of the structure of possible proofs. Another contentious point: Brouwer's 'seeing with the inner eye' included the 'intuitive' insight of God. Although this is never used in mathematics, there obviously is no measurable limit on what one can claim to be an 'intuitive insight'. The fallibility of the human mind is an additional weakness. The natural sciences, although their results may at first seem less certain, appear nevertheless very likely correct in suggesting that the human mind itself is subject to 'laws of nature' and so, through a complete turn of events undermine the primary position of the intuitive mind. Van Stigt's book will be the decisive text on Brouwer's intuitionism for many years to come, but there is still room for a critical discussion and comparative study of Brouwer's philosophy. This should perhaps be the subject of a new book.

References

[1] L.E.J. Brouwer. *Over de Grondslagen der Wiskunde,* D. van Dalen (editor). MC Varia, Vol. 1, Mathematisch Centrum, Amsterdam, 1981.

[2] D. van Dalen (editor). *Brouwer's Cambridge Lectures on Intuitionism.* Cambridge University Press, 1981.

[3] C. Smoryński. *Hilbert's Programme.* CWI Quarterly, Vol. 1, No. 4, 1988, 3–59.