Dreaming and Creativity in Mathematics

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How does one solve hard problems? and How does one teach it?

- The step-by-step process described to solve exercises does not work, and the failure causes frustration and the "I'm not good at math" response. [Word problems are always seen as the hardest part of a course].
- The process that I discovered using, without any training in "solving problems," was not at all like the linear, step-by-step process above.
- "Hard problems" are never solved by sitting long hours at a desk.

Goal and Purpose of Talk

- To better understand the successes and failures of the creative process in a mathematical context.
- Usually, only instances of success of the process are related. You can usually say what happened when it worked. Need to understand why one may have failed to learn from the attempt.
- One would also like to teach this process and help others to access the creative problem-solving process. Instead of assuming that eventually the student will figure it out (when you cannot describe "it").
- A nice example where nonlinear dynamics language makes sense as the actual successful process appears to be nonlinear where "dreaming" plays a role.

Starting Point

- Will be looking at the solving of "hard" problems – where a frontal direct (analytical) approach is not likely to succeed.
- These problems require a process that several (including Poincaré, 1915 and Hadamard, 1945) have described. The early work included little books by

Wallas (1926) The Art of Thought

and

Beveridge (1951) The Art of Scientific Investigation.

These were clearly used for many years as training books – not today.

Discoveries while doing the research

1) The creative process in this context is very closely related to play in children. Free play and direct experience of the mysterious and unknown form a basis for creative responses to life. :

In the creative perceptions of poet and child we are close to the biology of thought itself – close, in fact to the ecology of imagination, in which the energies of the body and mind as in a unit, an ecosystem, and the energies of nature combine in a mutual endeavor to adapt to nature, to culture and to the societies devised by man to embody culture.

Edith Cobb, 1977, The Ecology of Imagination in Child-hood.

The Universality of the Creative Process

2) Artists, writers, inventors, scientists, and mathematicians all tap the same source for the creativity they need. Differences lie in the nature of the problems solved or resolved, and the nature of the preparation for expressing the solution.

3) All recognize when the solution was "inspired" or "elegant" – the source of the insight is sometimes difficult to identify.

And more

4) The creative process itself has been a mystery for a long time – one can describe the results

I decided that is was not wisdom that enabled [poets] to write their poetry, but a kind of instinct or inspiration, such as you find with seers and prophets who deliver all their sublime messages without knowing in the least what they mean.

Socrates (469 BCE-399 BCE) found in Apology, sct 21 by Plato.

The Birth of the Muses

- *Zeus had brought the world into being, and the gods beheld in mute wonder the magnificence that lay before them. But, Zeus asked, is not something wanting? And the gods replied yes, one thing was wanting: the world lacked a voice whereby all this wonder could be expressed in words and music. In order for such a voice to sound there was a need for a new kind of divine beings – whereupon the Muses sprang into existence as the children of Zeus and Mnemosyne, goddess of memory.*

Pindar (518-438 BCE), "Hymn to Zeus"

Path to hearing the creative "voice"

- Traditionally begins with an invitation:

O Muse, Precious one sing to me,
Thy inspiration a prelude for my own song
Send a breeze from Your groves
Inspire my heart and mind
O Wise Kalliopia
Leader of the golden muses
And you too, wise initiator into mysteries,
Apollo, Son of Leto,
Be at hand, blessing me.

Mesomedes of Crete, 2nd c. C.E. "Hymn to the Muses"

More discoveries

5) Scientific curiosity is a fundamental aspect to the ability to hear the Creative Voice:

The most beautiful thing we can experience is the mysterious. It is the source of all true art and all science. He to whom this emotion is a stranger, who can no longer pause to wonder and stand rapt in awe, is as good as dead: his eyes are closed.

Albert Einstein from Mein Weltbilt (1931)

More musings

6) Why does my Muse only speak when she is unhappy? She does not, I only listen when I am unhappy. When I am happy I live and despise writing, For my Muse, this cannot be but dispiriting.

Stevie Smith (1902-1971), British poet and novelist.

7) May, 1994, identified creative people as those who can live with (periodic) anxiety and difficulties in exchange for the gift of "divine madness". The anxiety associated with the need to solve the problem is central to this process.

Problems with the word "creative"

- I discovered a large literature on creativity and how to foster it in different situations. They can all be divided into two piles: those where dreams and dreaming are mentioned, and where it is not. This is also often seen as using big C (dreams mentioned) and little c (no dreams).
- In the context of the Muses, it can be seen as those where an external agent is seen to be involved – or not.
- This can also be seen as to whether the subconscious aspects of thought are needed or not (related to the determination if the problem is hard or not).

Importance of "Dreaming"

- Dreaming is when the "codes of disciplined reasoning" are suspended. There are no rules of logical contradiction, spelling, pronunciation. (Daydreams, flights of thought, and dreams in sleep)
- Ordered, disciplined thought demands that one follows a set of rules to help one stay in a single frame of reference – to "stay focused"
- In a dream, one can shift from one matrix to another – often keeping both in mind at once – "we constantly bisociate in a passive way" – contradictions are ok.

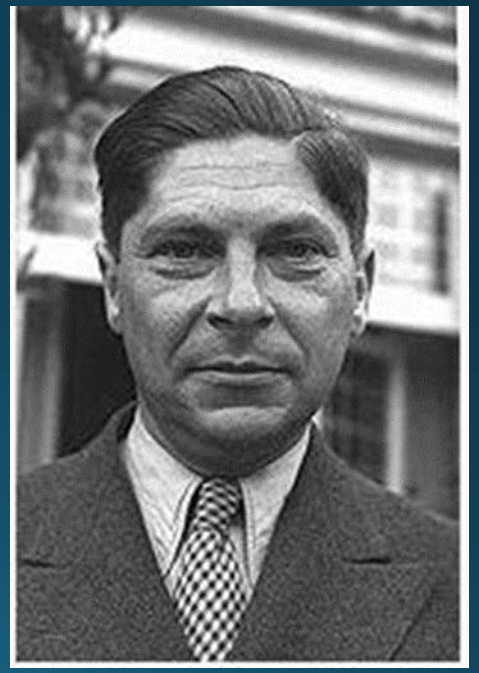
Difficulty with having dreaming involved

- Dreaming involves symbolic imagery – there can be difficulty in understanding what the subconscious has found.
- Communication between the unconscious (sleeping) state and conscious (awake) state can be difficult. Communication happens upon waking, when going to sleep, daydreams, …
- Remembering the insight discovered subconsciously can be difficult.
- Insight in dreams needs to be verified ("precised" in Hadamard's terms). It is not always correct – may have correct aspects or approach.

Arthur Koestler's (1964) mathematical description of the the creative process

Born in Budapest, educated in Austria, 43 years in Great Britain as a political commentator who wrote novels, essays, memoirs and biographies, and lectured widely.

He had a very controversial personal life (see wikipedia)



The Act of Creation was started in 1949 and published 15 years later (in mind a long time).

Koestler's basic idea

I have coined the term 'bisociation' in order to make a distinction between the routine [linear] skills of thinking on a single 'plane,' as it were, and the creative act, which, as I shall try to show, always operated on more than one plane. The former may be called single-minded, the latter a double-minded, transitory state of unstable equilibrium where the balance of emotion and thought is disturbed.

- Arthur Koestler, The Act of Creation, p. 34-35

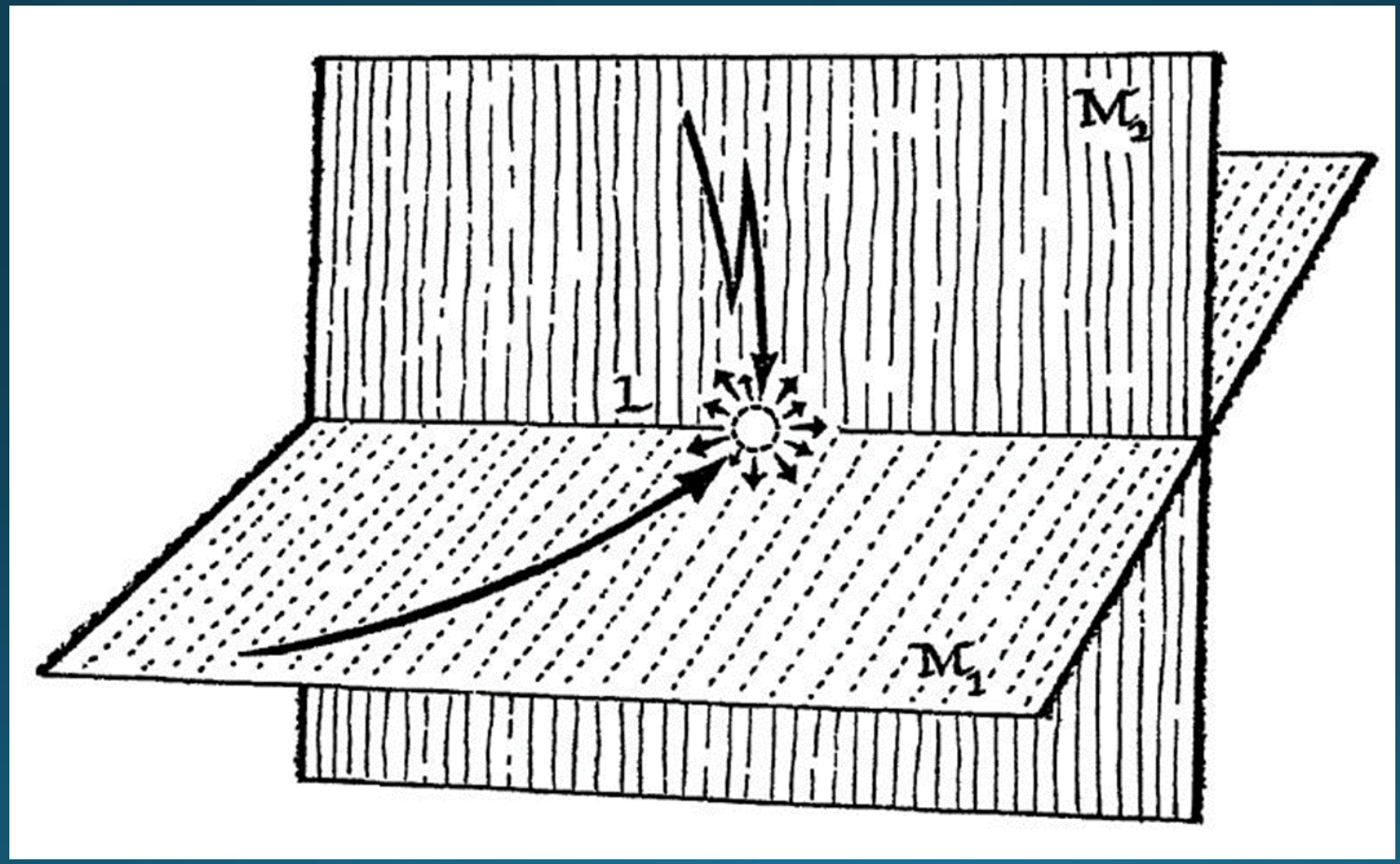
Bisociation

- Koestler gives many examples where jokes are effective because of this bisociation. Analogies are other ways in which useful ways of thinking (matrices) can be created.

Jokes:

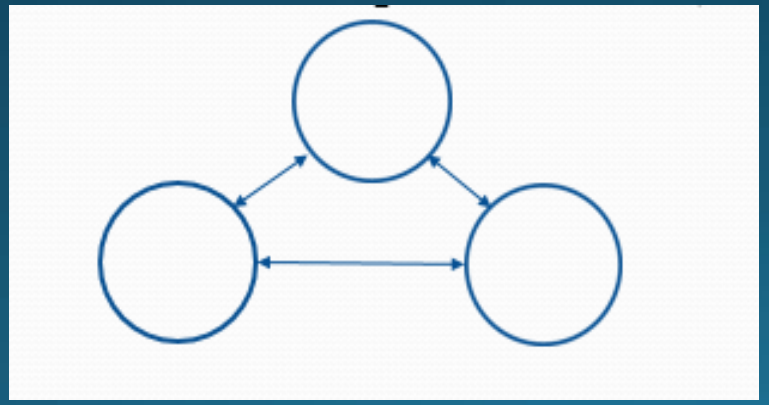
- Atheists don't solve exponential equations because they don't believe in higher powers.
- Some people's noses and feet are built backwards: their feet smell and their noses run.
- When the cannibal showed up late to the conference luncheon, they gave him the cold shoulder.

Bisociation – two frames of reference resulting in humor

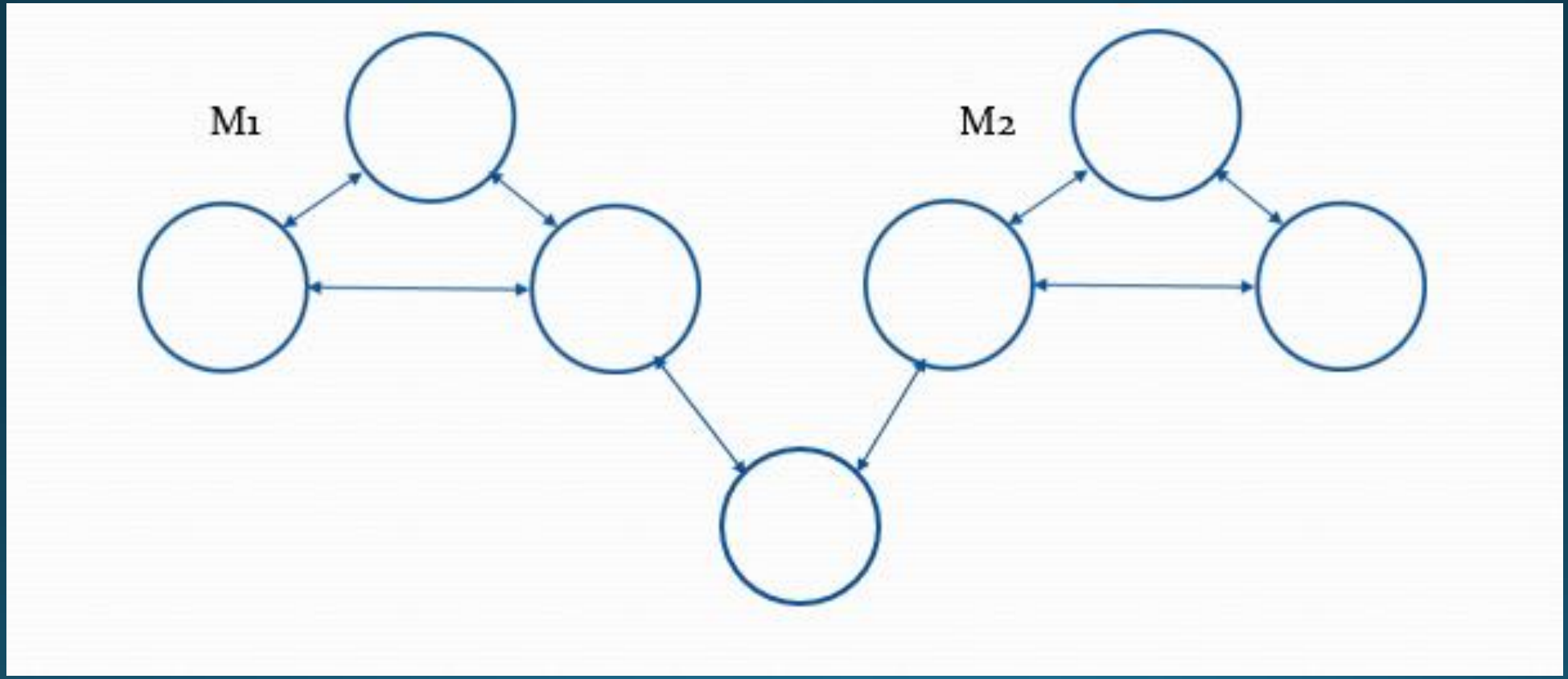


A conceptual model

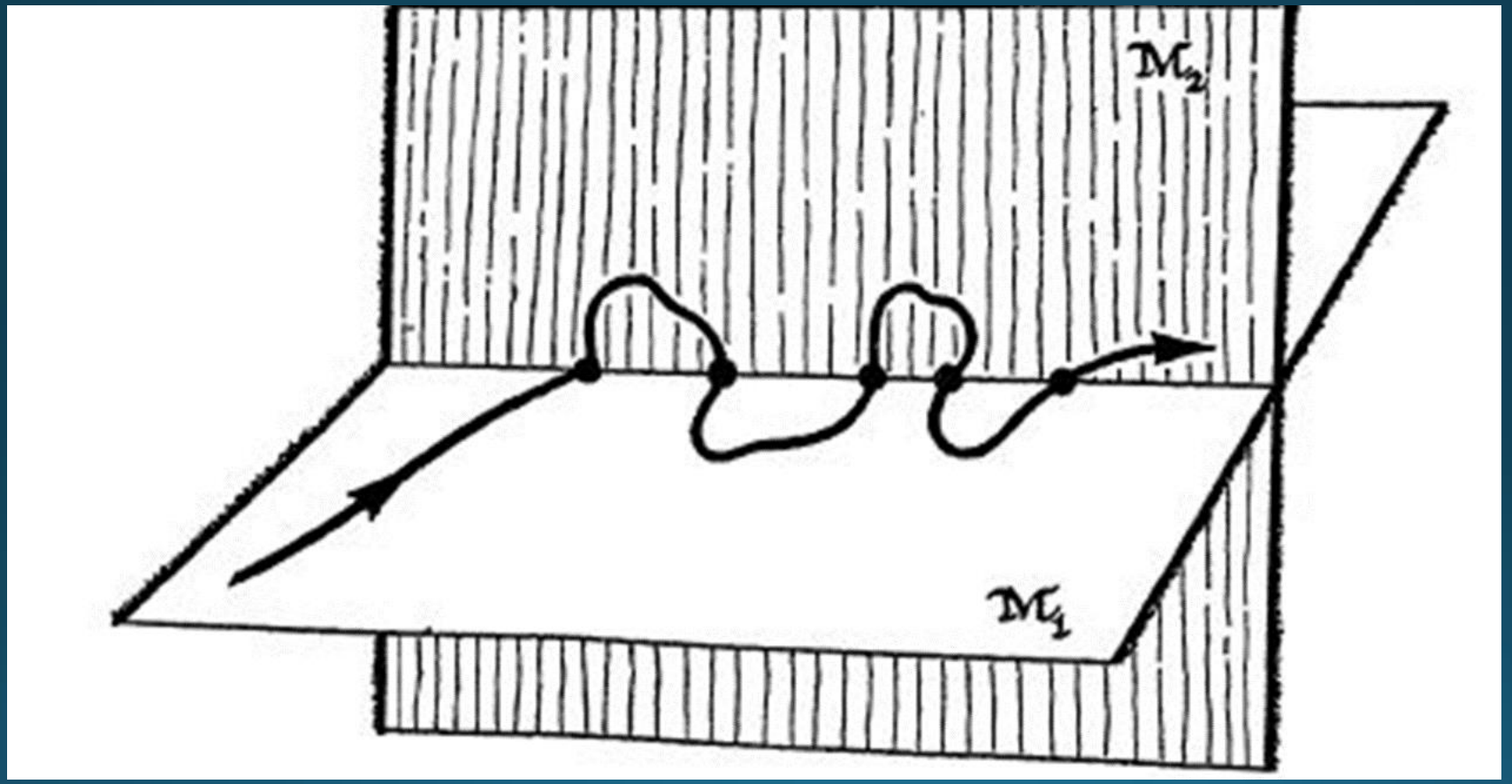
- Building a model to put the words into mathematical form.
- Provides a way to explore the ideas presented.
- We will associate a frame of reference ("matrix") as an ergodic Markov chain. In this situation, the frame is stable in one cannot escape the chain (collection of states).



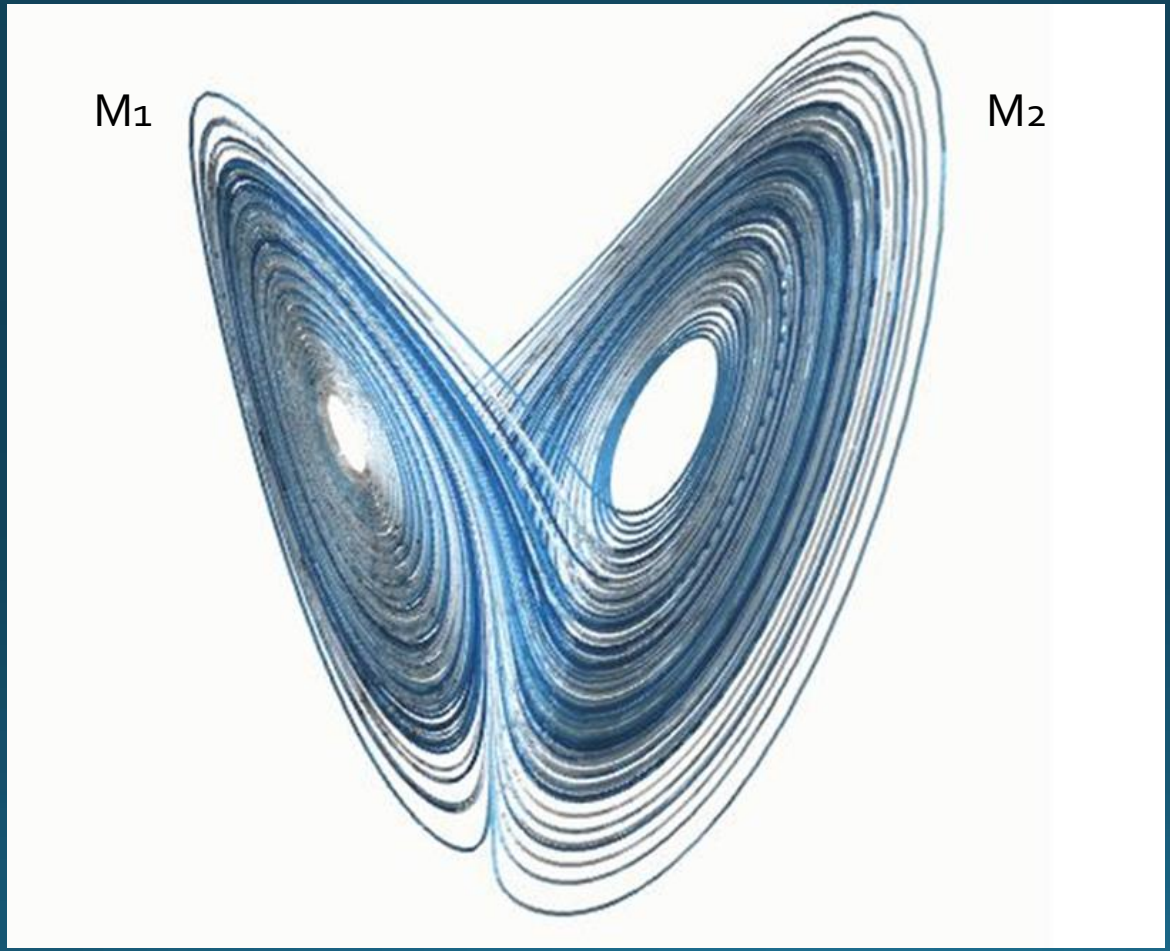
Two subchains and an extra connecting state



Thinking oscillating between frames of reference ("matrices")



Bisociation in the Lorenz system



The role of the "connecting" state

- The model suggests that a state that communicates with both frames of reference is important. In humor, that could be a word with two meanings or a pun.
- In solving hard problems one popular approach is the idea of TRIZ, theory of inventive problem solving (see wiki). One idea here is to identify the contradiction present in the ability to solve a hard problem. For instance, "how can the best quality product cost the least?" The result of this process can be seen as defining the two different frames of thought or the "matrix of contradictions". They do not, however have a way to generate the key connection.

Analogy building

- Analogies are frames having important properties in common.
- The analogy often does not carry the same level of complexity as the system of interest.
- Often the problem can be solved in some form in the analogous situation.
- Bouncing back and forth between the system of study and the analogy can mimic this process of bisociation.
- Literature for use of analogies in problem solving includes Gick & Holyoak, 1980 (and the 1615 papers that have referenced them) and Novik & Holyoak, 1991, but emphasize "analytical" problem solving and not the creativity needed to solve hard problems.

Mathematical modeling as a model of the creative process

- Has the advantage that it involves analogy-building.
- Tools that are needed are beyond the application of known facts and their logical extention
  - *Is by logic that we prove but by intuition we discover*
    - *Poicaré, 1913*
- 35 years of teaching computational modeling and observing students – in a class where a "hard" problem was required to be solved to pass.
- Discussed in S. Merrill (2003), "Perchance to dream", ICTMA 11.

What remains is to describe how to prepare a problem for solution

- There are two key points involving the subconscious:

  1) The problem must be defined in such a way that the subconscious begins work (in the background and at night aka "the committee of sleep")

  This involves the presence of stress – the imperative need to solve the problem – to pass a class, finish a dissertation, get tenure, …

  2) A symbolic language for communication developed so that a solved problem has a way to emerge to the conscious mind. This is done through building analogies, and considering the form that an answer would take.

Steps identified from Early Work

1) Preparation – three aspects that must be in place

2) Incubation – sleeping & waking focus on the problem

3) Illumination – realization that "a solution" may have been found. In a dream, upon waking, or popping into your head.

4) Verification – testing of the proposed solution.

Preparation – 3 parts

1) Preparation in the discipline of the art – without technique, the insight cannot be used. One must develop facility in the art until one no longer thinks about it (or needs to look things up) – until you can do it in your sleep. "When skill hides itself in the unconscious, it reveals the unconscious." Nachmanovitch, 1990.
2) Preparation in the discipline (habit) of listening to the voice – practice in remembering and evaluating your thoughts and dreams.
3) Preparation through immersion in the problem – you know when it is enough when, even waking, you realize that you are thinking about the problem.

Incubation (Beveridge, 1951)

1) Prolonged contemplation of the problem for days

2) Freedom from interruption and other interests competing for attention.

3) Intuitive thoughts are more likely to come during a period of apparent (mindless) activity.

4) New ideas vanish quickly – they must be captured.

Illumination – as a result of non-conscious brain activity

- Interesting note, Leibniz was probably the first to note that the unconscious records a more complete record of the day than we realize. Details of a problem are more complete in the unconscious.
- REM dreams have been identified as most useful in creative contexts, but mathematicians and scientists, hypnangogic (when falling asleep), hypnopompic (when waking), as well as daydreams are most useful.
- There is a difference between when the work is done and how it is communicated.

Verification

- As noted by Hadamard, there is the need to verify insight from dreams – they are not always correct. My experience is that even though not correct, often the dream does provide a breakthrough.
- Good example – Lorenzo's Oil – Universal Studios, 1992.

In Summary

- It is possible to employ the Committee of Sleep in a way to assist in the solution of hard problems. The steps are clearly identified and can be taught (at least after the fact to recognize what happened).
- By recognizing what is involved, one can return to hear the creative voice again.

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