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The State of the Science of Nonlinear Dynamics in 1963

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Abstract: *When Edward Lorenz published his paper in 1963 on the deterministic nonperiodic solutions present in what is now called the "Lorenz Equations," he did so in the context of a rich theory of nonlinear dynamics beginning with the work of Poincaré. Here we trace the influences present in his work as well as provide a snapshot of the nonlinear world in 1963 to explain why his paper had such a pervasive effect on the science of nonlinear dynamics and its applications. In doing this, we also discover the critical timing of the development of the computer in this effort.*

Key Words: Lorenz, Poincare, history, nonlinear dynamics

INTRODUCTION

In Lorenz (1963), the author, through his references and the style of exposition, acknowledged three primary mathematical sources that were central to his study of nonlinear dynamics of solutions to his model. The first was Henri Poincaré, a Frenchman who worked at the end of the 19th century and beginning of the 20th. Poincaré's papers, essays, and books motivated the study of what appeared to be (and were) challenges to the dogma that the determinism of Newtonian mechanics ruled all things. He also established the field of topology and showed its power through studies of nonlinear systems. The second was G. D. Birkhoff, who took Poincaré's ideas and extended them in new directions, and established the discipline of "dynamical systems." It is said that Birkhoff was the first American-trained great mathematician. The last was the book by Nemytskii and Stepanov, written in 1949 in Russian, which provided the specific mathematical language of Lorenz's remarkable paper (an English translation having appeared in 1960).

In this paper, those influences are traced to describe a remarkably international effort to describe the unusual dynamics in examples arising in celestial mechanics, electronics, hydrodynamics, weather, and ecology. There are some marvelous and valuable surveys of this effort and time. Especially noted are Lorenz (1995), Holmes (2005), and Smale (1998). There were many players in this effort, and each has a unique and valuable viewpoint as to the state of this system in 1963 – in preparation for the explosion which was to

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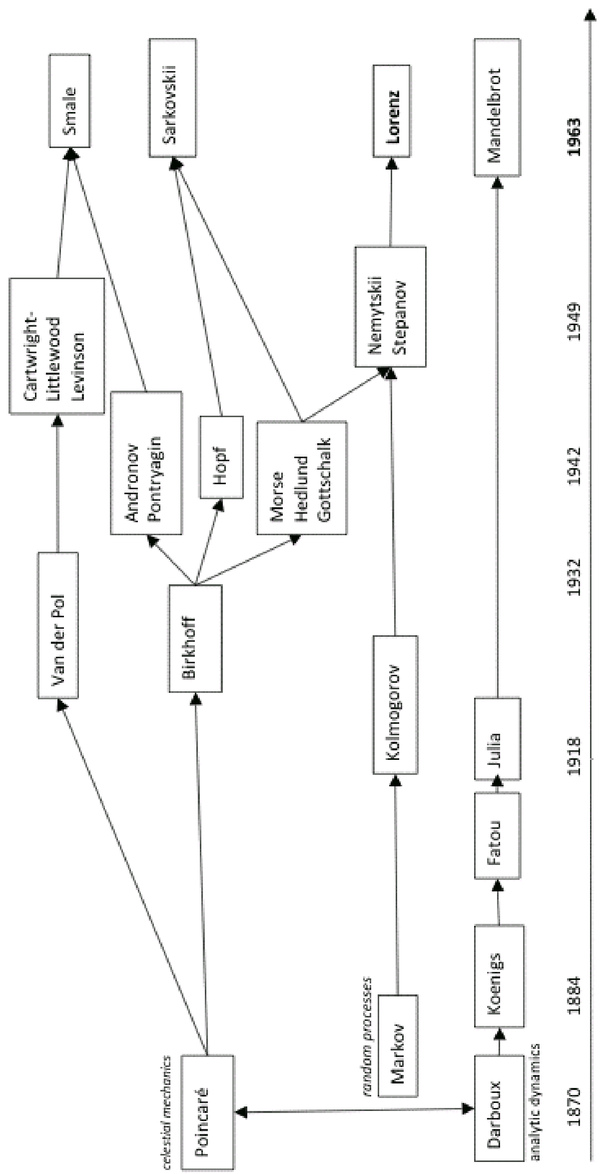

Fig 1. Timeline of the development of nonlinear dynamics from 1863-1963.

happen. There is also another parallel story which will be described, and that is the development of the computer. The availability of the digital computer has proven indispensible both to the discovery of nonlinear phenomena and its exploration by later investigators (and artists).

A MAP OF THE DEVELOPMENT OF THE SCIENCE OF NONLINEAR DYNAMICS

In Fig. 1, the development of ideas from 1890-1963 is traced (dependencies were determined by examining the references and discussions of the authors in their fundamental papers and books). In this manner, the time course of the development of the ideas can be determined. Most of the primary ideas present in nonlinear dynamics today can be seen to be in place in 1963. The idea of sensitive dependence on initial conditions was from Poincaré, recognized through his description of the importance of hyperbolic structures. The presence of what we now see is chaos is often demonstrated by showing the presence of the (Smale) horseshoe – discovered by Smale through study of the work on the forced van der Pol equation by Cartwright and Littlewood (1945), and Levinson (1949; Coddington & Levinson, 1955). Hopf had described the bifurcation of solutions in 1942 broadening the study of systems with a parameter to include the bifurcation diagram. The study of iterated functions by Sarkovskii described the path of bifurcations to chaos in an iterated system, seen later by May (in the discrete logistic equation) and then Li and Yorke who coined the word “chaos” to describe the nature of the behavior of the solutions when parameters were in the right range. Fractals had been recognized by Mandelbrot first in 1963, building on the ideas of Fatou (1917, 1918) and Julia (1918) from the period after WWI. His graphical study of those phenomena began at that time – eventually made possible by access to unlimited computing resources at IBM.

Figure 1 greatly simplifies the whole picture, but it does indicate the world-wide nature of the work and the energy present in that effort. The role of the computer and the resulting graphics on highlighting the importance of Lorenz’s work (and later that of Mandelbrot, 1977) cannot be overestimated.

Lorenz reports that the simulations he reported were computed on a Royal McBee LGP-30, a single-user desk computer (it took up the whole desk) made by Royal Typewriter. It weighed 800 pounds and used of over a hundred electronic tubes and over a thousand diodes in its circuit boards. It is probable that the tubes themselves acted in a chaotic manner as shown by van der Pol and van der Mark, (1927). The main memory was 4096 words. The computer is shown in Figs. 2 and 3 from an advertisement and a brochure. The story of this computer, the first “personal” computer, is an important related story.

The LGP-30 was designed by Stan Frankel while at Cal Tech in the early 1950s. A summary of his career can be found in Liebson (2006). Frankel was a post doctoral fellow with Oppenheimer at Los Alamos during the Manhattan Project. In that position, he interacted with all of the famous scientists present in that place. He also was central to the effort of trying to accomplish the

Fig. 2. The Royal McBee LGP-30. Top part of the ad from the March 1957 issue of *IRE Transactions on Electronic Computers*.

to test the ideas of Teller in the development of the hydrogen bomb. At that time, in 1945, the ENIAC was becoming available for use. Before that, the most powerful calculation aid in T division was an IBM card tabulation machine. After the war, developing a simple personal computer for scientific use was an obsession for Frankel. His development of the MINAC, marketed as the LGP-30 was described in 1957 (Frankel, 1957). The advertisement for the LGP-30 in that issue was a full page, with a drawing shown in Fig. 2.

DISCUSSION

In some sense, Lorenz's contribution was said to discover "sensitive dependence on initial conditions" in a simplified weather model. That result had been known by Poincaré by at least 1914 in *Science and Method*:

> If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. But even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation *approximately*. If that enabled us to predict the succeeding situation with *the same approximation*, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that

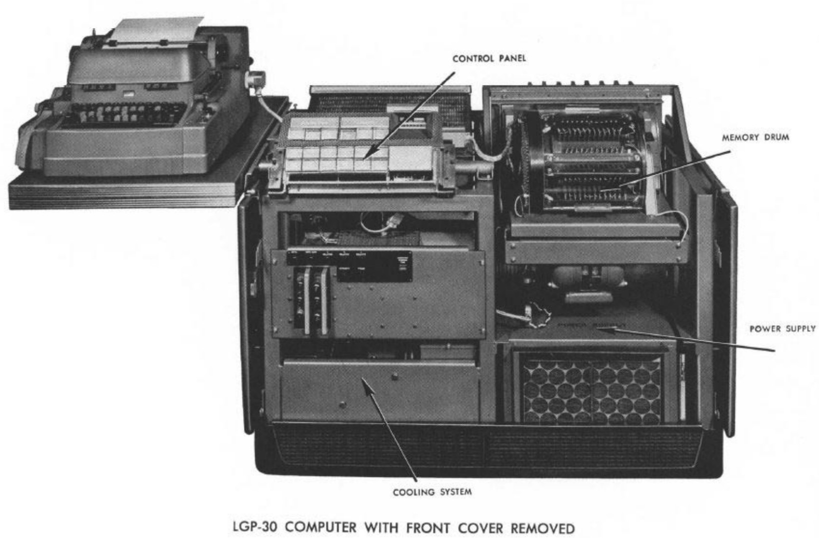

Fig. 3. Photo from inside the LGP-30 1957 brochure.

> small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.

In another sense, maybe the contribution was important because it was an equation describing (in a simplified way) a real physical system. But in 1945, Cartwright and Littlewood presented evidence of chaotic motion in a forced van de Pol equation which modeled the operation of a radio tube. This result seemed to come at a time when the world was not yet ready or was unable to see its significance. It needs to be noted that observations of hydrodynamical systems had been known to display irregular behavior that were not seen to repeat over long observation. Lorenz notes two studies from the 1950s involving a rotating basin, but others involved heat applied to the system. In all of the cases above, the systems are forced, in that external energy is added to a system resulting in complicated behavior from the system. In every case, the systems behavior was greatly simplified (or was trivial) when the energy is removed.

Lorenz's contribution was not that the weather problem had been reduced to a system of ordinary differential equations, as his colleague Saltzman had presented a way to do this in 1962 and Lorenz himself had been working in the direction of simplifying the dynamics for several years. The contribution is clearly that: (a) the system was solved numerically and the graphical presentation of the result was compelling; (b) the system presented was autonomous – not requiring external energy to display its behavior; (c) Lorenz had clearly read the works of those before and presented the results in the context of current mathematics (even though presenting the result in the Journal of Atmospheric Sciences); (d) The work was used to suggest that there may be many simple nonlinear systems that may show interesting behavior – and that turned out to be the case.

Any science starts with careful description of the phenomena. The development of science then goes through a stage of theory-building to create and test the understanding of what has been described. When this is complete, application of the results can proceed. Lorenz presented what turned out to be the initial simple example of the phenomena predicted by others, thereby starting what is now the science of nonlinear dynamics.

SUMMARY

It is clear that there was a rich framework in place for Lorenz to describe the interesting behavior in 1963. His work set off the search for more examples displaying complicated behavior as well as further study of his system as soon as computational tools became more readily available. By 1982, it was possible for Sparrow to write a complete monograph on the topic of the Lorenz equations.

The study of this system opened up the experimental side of mathematics and led to the development of nonlinear dynamics by encouraging young quantitative scientists to explore and apply the results and insights of the exciting "new" field. Similar excitement was found in the fractal pictures of Mandelbrot and the behavior of iterated maps. The next step was to apply this knowledge.

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