Calculus for the Biological Sciences

Lecture Notes – Introduction

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Outline

1 Introduction

- Why Math 1410 is needed for Biologists
- Mathematical Models

Math 1410:Introduction

- Biology is rapidly expanding - more quantitative analysis of the data
- Mathematics and computers are more important
- This course in Calculus for Biology
  - Emphasis on mathematical modeling of biological systems
  - Lecture notes show how Calculus naturally arises in biological examples
  - Begin with a biological model
  - Mathematical theory required to analyze the biological problem
- Use real or realistic examples
- Computer labs aid the more complicated models

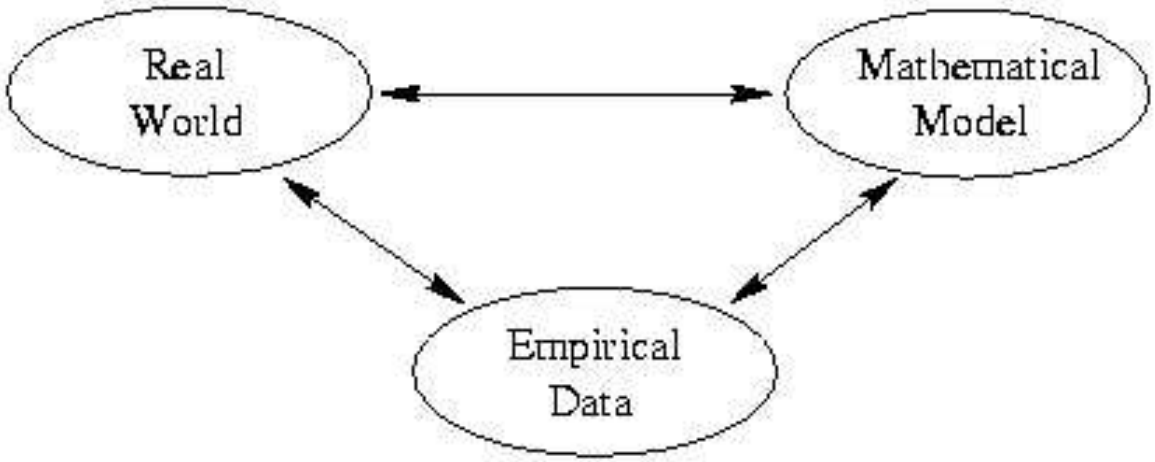
Math 1410:Introduction — Mathematical Biology

Mathematical Biology

- Mathematical tools
  - Better qualitative and quantitative understanding of biological problems
  - Suggest alternate possibilities
  - Reject inconsistent ideas
- Biological problems
  - Often stretch mathematical techniques
  - Illustrate mathematical tools well
  - Build intuition for problem techniques

Math 1410:Introduction — Mathematical Model

So what is a mathematical model?



Math 1410:Introduction — Mathematical Model

- A **mathematical model** is a representation of a real system
- It is simple in design
- It exhibits the basic properties of the real system
- The model should be testable against empirical data
- Comparisons of the model to the real system should lead to improved mathematical models
- The model may suggest improved experiments

Introduction – Example – Diabetes mellitus

Biological Information

- Metabolic disease characterized by too much sugar in the blood and urine
- β-cells in the pancreas release insulin in response to rises in levels of glucose in the blood
- Stores energy as glycogen in the liver
- Juvenile diabetes (Type I) - failure of the β-cells to release insulin to blood glucose levels – an autoimmune response killing β-cells
- Adult onset diabetes (Type II) results in insulin resistance – cells fail to use insulin properly

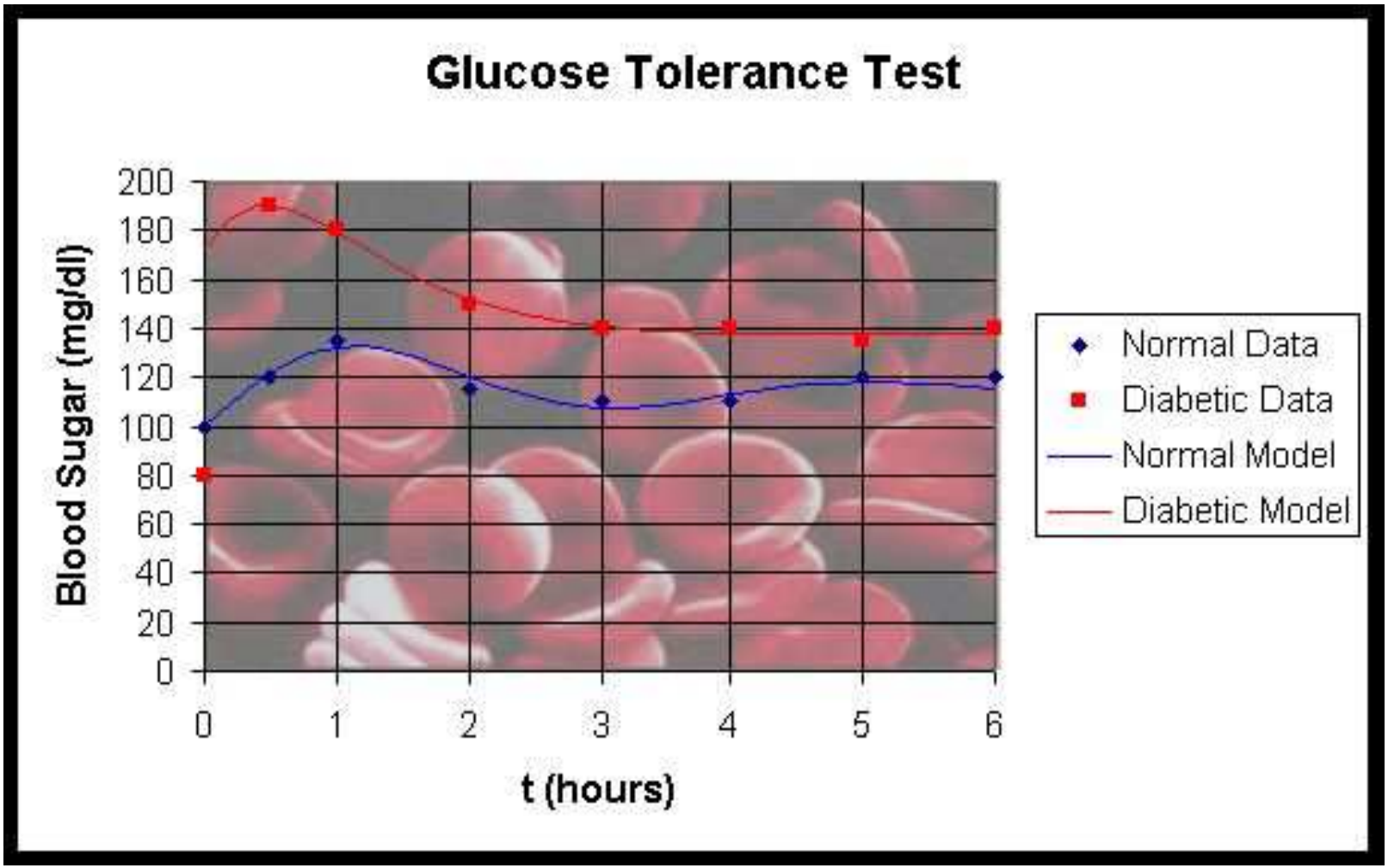
Diabetes mellitus – Ackerman Model

Ackerman Model for Diabetes

- Glucose Tolerance Test (GTT)
  - Subject fasts for 12 hours
  - Given a large quantity of glucose
  - Blood sampled regularly for 4-6 hours
- Mathematical Model
  - 2-Component model - Blood glucose and insulin levels
  - Linear system of differential equations (Damped harmonic oscillator)
  - Simple solution with exponentials and trig functions
  - Solution fit to data
  - Parameter values indicate health of subject

Ackerman Model for Diabetes

Glucose Tolerance Test



Introduction – Example – Predator-Prey Model

Predator-Prey Model

Thanks to Tom and Pat Leeson

Example – Predator-Prey Model

Predator-Prey Model

- In the early 20^{th} century, Sir Ronald Ross used mathematical modeling to show that malaria could be eliminated without the total eradication of mosquitoes
- A. J. Lotka [1] first studied the population dynamics of predator-prey interactions
- Studies of Vito Volterra on fishing in the Adriatic Sea in 1924 showed value of a simple model for equilibrium analysis
- **Predator-Prey models** are often called **Lotka-Volterra models**
- Widely used by biologists – however, significant flaws in the mathematical understanding often lead to poor conclusions

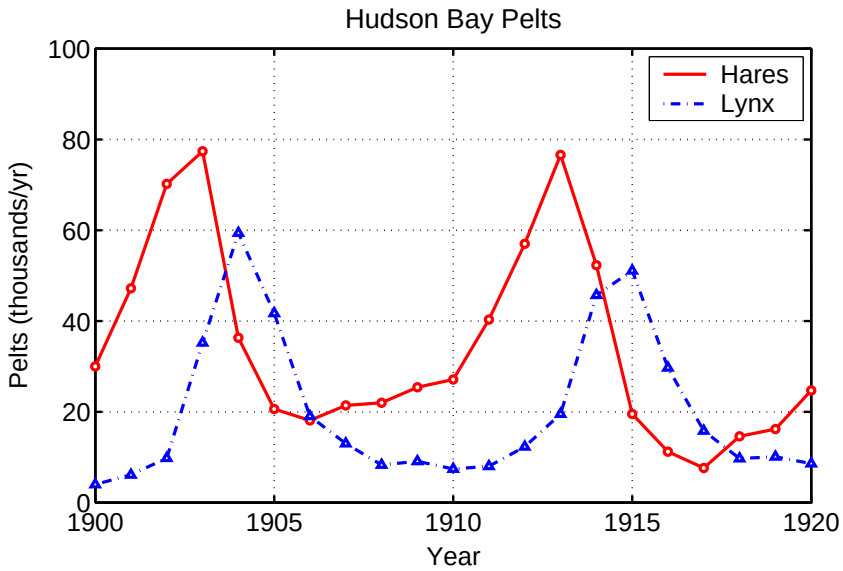
[1] A. J. Lotka (1912), Evolution in Discontinuous Systems, *Journal of the Washington Academy of Sciences*, **2**, pp.2, 49, 66

Example – Predator-Prey Model

Classic Lynx-Hare Data

- Records of the Hudson Bay Company show that the pelts of the lynx and hares seemed to oscillate with a fairly regular period
- Simple ecological system, as the lynx is a very specialized predator that primarily feeds on snowshoe hares
- Books often cherry-pick to show limited data - Model fails badly over the complete data set
- We'll examine this model late in the semester

Example – Predator-Prey Model



- Graph shows a clear correlation between the populations of lynx and hares
- Rapid rise in the population of the hares is followed by a rapid rise in the lynx population
- Next the hare population plummets, which is followed by lynx population plummeting