Interesting Formulae

Trigonometric Identities

$$\sin\alpha = \left(e^{i\alpha} - e^{-i\alpha}\right)/(2i) \qquad \sinh\alpha = \left(e^{\alpha} - e^{-\alpha}\right)/2$$

$$\cos\alpha = \left(e^{i\alpha} + e^{-i\alpha}\right)/2 \qquad \cosh\alpha = \left(e^{\alpha} + e^{-\alpha}\right)/2$$

$$\cos^2\alpha + \sin^2\alpha = 1 \qquad \cosh^2\alpha - \sinh^2\alpha = 1$$

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta) \qquad 2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

$$2\sin\alpha\cos\beta = \sin(\alpha-\beta) + \sin(\alpha+\beta)$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha \qquad \sinh(2\alpha) = 2\sinh\alpha\cosh\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha \qquad \cosh(2\alpha) = \cosh^2\alpha + \sinh^2\alpha$$

Indefinite Integrals

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int x\sin(ax)\,dx = -\frac{x}{a}\cos(ax) + \frac{1}{a^2}\sin(ax) \qquad \int x\cos(ax)\,dx = \frac{x}{a}\sin(ax) + \frac{1}{a^2}\cos(ax)$$

$$\int x^n\sin(ax)\,dx = -\frac{x^n}{a}\cos(ax) + \frac{n}{a}\int x^{n-1}\cos(ax)\,dx, \quad n > 0$$

$$\int x^n\cos(ax)\,dx = \frac{x^n}{a}\sin(ax) - \frac{n}{a}\int x^{n-1}\sin(ax)\,dx, \quad n > 0$$

$$\int \sin(ax)\cos(bx)\,dx = -\frac{1}{2}\left\{\frac{\cos[(a-b)x]}{a-b} + \frac{\cos[(a+b)x]}{a+b}\right\}, \quad a^2 \neq b^2$$

$$\int \cos(ax)\cos(bx)\,dx = \frac{1}{2}\left\{\frac{\sin[(a-b)x]}{a-b} + \frac{\sin[(a+b)x]}{a+b}\right\}, \quad a^2 \neq b^2$$

$$\int \sin(ax)\sin(bx)\,dx = \frac{1}{2}\left\{\frac{\sin[(a-b)x]}{a-b} - \frac{\sin[(a+b)x]}{a+b}\right\}, \quad a^2 \neq b^2$$

$$\int \cos^2(ax)\,dx = \frac{1}{2}\left[x + \frac{\sin(2ax)}{2a}\right] \qquad \int \sin^2(ax)\,dx = \frac{1}{2}\left[x - \frac{\sin(2ax)}{2a}\right]$$

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx) - b\cos(bx)\right]$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos(bx) + b\sin(bx)\right]$$

$$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

Definite Integrals

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \quad \mathrm{Re}(a) \geq 0 \qquad \int_0^\infty x^2 e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}, \quad \mathrm{Re}(a) > 0$$

$$\int_0^\infty \cos(px)e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}e^{-p^2/(4a)}, \quad \mathrm{Re}(a) \geq 0 \qquad \int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$$

Formal Fourier Facts

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right\}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(z) \cos \frac{n\pi z}{L}\, dz\ , \qquad b_n = \frac{1}{L} \int_{-L}^{L} f(z) \sin \frac{n\pi z}{L}\, dz$$

$$\frac{1}{L} \int_{-L}^{L} |f(z)|^2\, dz = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} \left\{ |a_n|^2 + |b_n|^2 \right\}$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{g}(\omega) e^{-i\omega t}\, d\omega\ , \qquad \hat{g}(\omega) = \int_{-\infty}^{\infty} g(t) e^{i\omega t}\, dt$$

Laplacian Operator

$$\Delta u = \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}$$

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{r^2 \sin\phi} \frac{\partial}{\partial \phi} \left(\sin\phi \frac{\partial u}{\partial \phi} \right) + \frac{1}{r^2 \sin^2 \phi} \frac{\partial^2 u}{\partial \theta^2}$$

(Spherical coordinates: $x = r\cos\theta\sin\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\phi$).

Div, Grad, Curl and All That

$$\nabla(a \cdot b) = (a \cdot \nabla) b + (b \cdot \nabla) a + a \times (\nabla \times b) + b \times (\nabla \times a)$$

$$\nabla \cdot (a \times b) = b \cdot (\nabla \times a) - a \cdot (\nabla \times b)$$

$$\nabla \times (a \times b) = a(\nabla \cdot b) - b(\nabla \cdot a) + (b \cdot \nabla) a - (a \cdot \nabla) b$$

$$\int_D \nabla \cdot F\, dx = \int_{\partial D} \hat{n} \cdot F\, dS$$

$$\int_R \nabla \times A \cdot \hat{n}\, dS = \oint_{\partial R} A \cdot dl$$

$$\int_D (v\Delta u - u\Delta v)\, dx = \int_{\partial D} [v(\hat{n} \cdot \nabla u) - u(\hat{n} \cdot \nabla v)]\, dS$$

$$\int_D \Delta u\, dx = \int_{\partial D} \hat{n} \cdot \nabla u\, dS$$

$$\int_D (v\Delta u + \nabla v \cdot \nabla u)\, dx = \int_{\partial D} v(\hat{n} \cdot \nabla u)\, dS$$