# Calculus for the Biological Sciences

## Exponential Functions

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# Exponential Functions

# Population Growth

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- Find the relative change for each year between 2000 and 2003.

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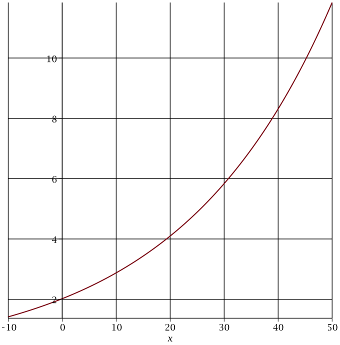
$$P = 2.020(1.036)^t.$$
- The population is an **exponential function** (with respect to $t$).
- 1.036 represents the factor by which the population grows each year. It is called the **growth factor**.

# Population Growth

Assuming that the formula holds for 50 years (since 2000).



# Elimination of Drug from the Body

**Problem 1.** When a patient is given medication, the drug enters the bloodstream. The rate at which the drug is metabolized and eliminated depends on the particular drug. For the antibiotic ampicillin, approximately 40% of the drug eliminated every hour. A typical dose of ampicillin is 250 mg. Suppose $Q = f(t)$, where $Q$ is the quantity of ampicillin, in mg, in the bloodstream at time $t$ hours since the drug was given. Find several initial values of $f(t)$.

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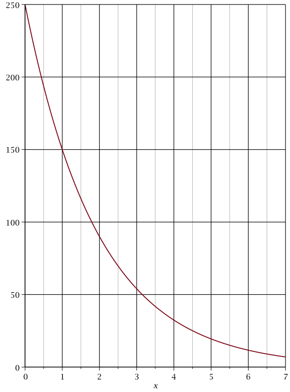

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- If $a > 1$, we have an **exponential growth**.
- If $0 < a < 1$, we have an **exponential decay**.
- $a = 1 + r$, where $r$ is the decimal representation of the percent rate of change.

# Comparison between Linear and Exponential Functions

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- An exponential function has a constant percent rate of change (relative rate of change).

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- Decreasing by 7 per minute.
- Increasing by 4% per minute.
- Decreasing by 6% per minute.

# Example

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- $$P_0 = 2503$$
- $$a^6 = 1.478$$
- $$a = 1.07$$

# Recognizing Data

**Definition**

The values of $t$ and $P$ in a table could from an exponential function $P = P_0a^t$ if ratios of $P$ values are constant for equally spaced $t$ values.

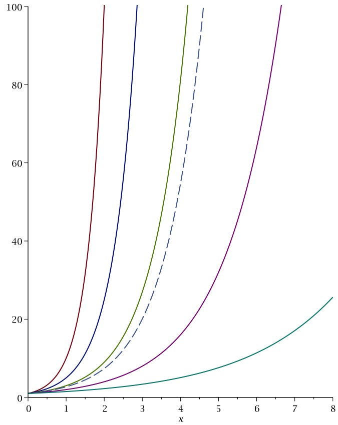
# Example

| x | f(x) |
|---|---|
| 0 | 16 |
| 1 | 24 |
| 2 | 36 |
| 3 | 54 |
| 4 | 81 |

| x | g(x) |
|---|---|
| 0 | 14 |
| 1 | 20 |
| 2 | 24 |
| 3 | 29 |
| 4 | 35 |

| x | h(x) |
|---|---|
| 0 | 5.3 |
| 1 | 6.5 |
| 2 | 7.7 |
| 3 | 8.9 |
| 4 | 10.1 |

# Families of exponential functions



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