Calculus for the Biological Sciences

Average rate of change and relative change

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Average rate of change

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10.06	9.95	9.93	9.92	9.86	9.85	9.84	9.79	9.70	9.77	9.74	9.69	9.58

Table: Men 100m World Records (Running)

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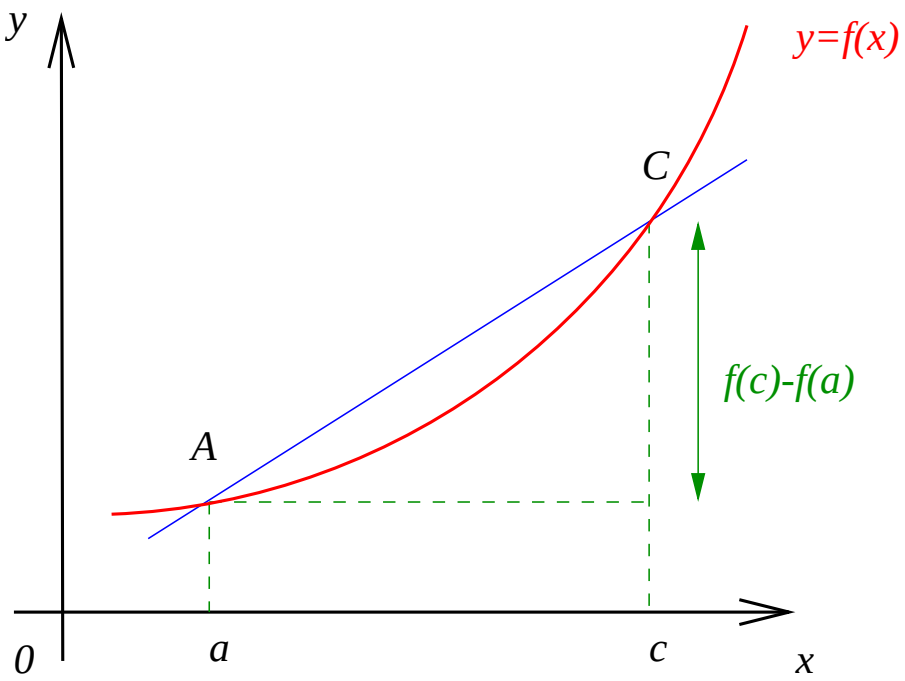
Average rate of change

Definition

If y is a function of t, so $y = f(t)$, then the **average rate of change** of y between $t = a$ and $t = b$ is

$$\frac{\Delta y}{\Delta t} = \frac{f(b) - f(a)}{b - a}.$$

Visualize Rate of Change



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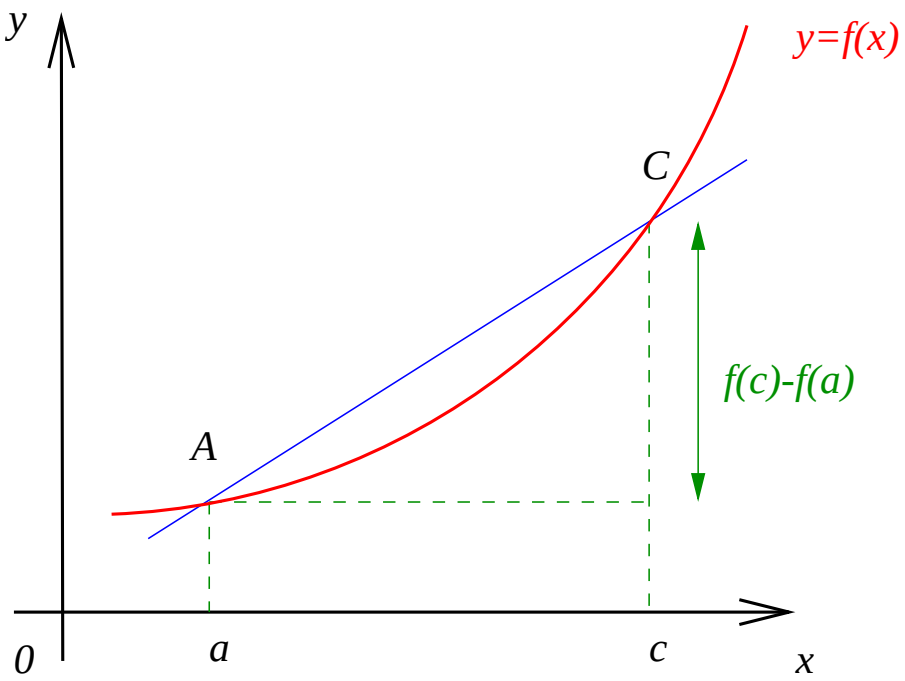
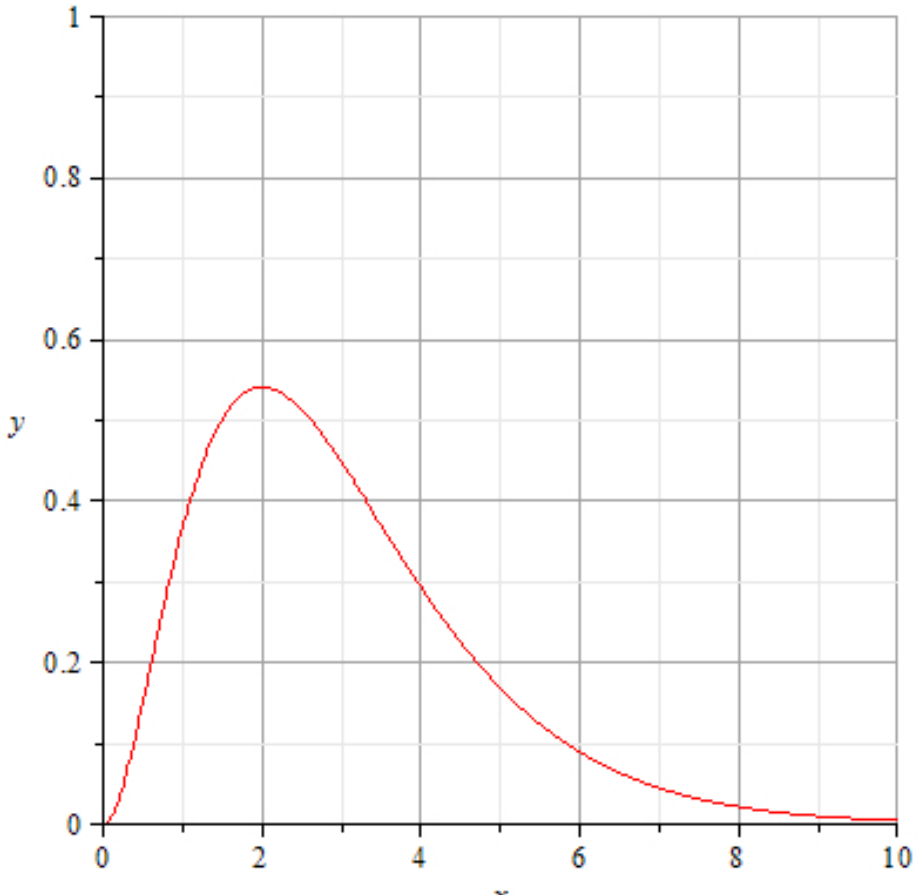


- The line passing through A and C is called the secant line between $x = a$ and $x = c$.
- The average rate of change is represented by the slope of the secant line.

Concavity

Definition

The graph of a function is **concave up** if it bends upward as we move from left to right; the graph is **concave down** if it bends downward.



Distance, Velocity, and Speed

Problem 1: A ball is thrown up in the air. The height of the ball above the ground is represented by the table. Find the rate of change and average rate of change in the first 3 seconds.

t (sec)	0	1	2	3	4	5	6
y (ft)	6	95	150	160	150	100	40

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- Speed is the magnitude of velocity.

Distance, Velocity, and Speed

Problem 2: Find the average velocity of the ball over the interval $t = 2$ and $t = 3$. Compare to the similar value over the interval $t = 3$ and $t = 4$. Explain the difference.

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y (ft)	6	95	150	160	150	100	40

Distance, Velocity, and Speed

Definition

$$\text{Average velocity} = \frac{\text{Change in distance}}{\text{Change in time}}$$

Relative change

- **Is a population increase of 30 a significant change?** If the number of graduate students in the math department, there are total 100 students, increases by 30 people, the graduate students would definitely notice. On the other hand, if the number of IU Bloomington students, there are total about 43,000 students, increases by 30 people, almost no one will notice.

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- To visualize the impact of the increase on the two different communities, we look at the change, 30, as a fraction of the initial population. This change is called the **relative change.**

Relative change

Definition

When a quantity P changes from P_0 to P_1, we define

$$\text{Relative change in } P = \frac{\text{Change in } P}{P_0} = \frac{P_1 - P_0}{P_0}$$

The relative change is a number, without unit. It is often expressed as a percentage.