# Calculus for the Biological Sciences

## Applications of functions to economics

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# The cost function

## Definition

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The **cost function**, $C(q)$, gives the total cost of producing a quantity $q$ of some good.

- More goods that are made, the higher the total cost.
- Costs of production can be separated into two parts:
  - The **fixed costs**: which incurred even if nothing is produced;
  - The **variable costs**: which depend on how many unit produced.

# Cost function: Example

A company makes radios. The factory and machinery needed to begin production are fixed costs. The cost of labor and raw materials are variable costs. Assume that the fix costs for this company is $24,000 and the variable cost are $7 per radio.

- $$\begin{aligned} \text{Total cost} &= \text{Fixed costs} + \text{Variable costs} \\ &= 24,000 + 7 \cdot \text{Number of radios} \\ &= 24,000 + 7q \end{aligned}$$

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- The variable cost for one additional unit is called the **marginal cost**
- For a linear cost function, the marginal cost is the rate of change, or the slope, of the cost function.

# Example

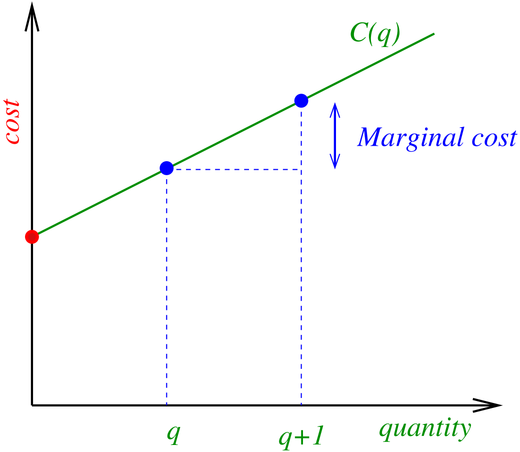


Figure : Cost function for the radio manufacture

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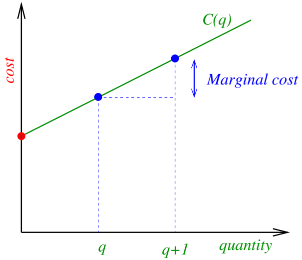


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If $C(q)$ is a linear cost function,

- Fixed cost is represented by the vertical intercept.

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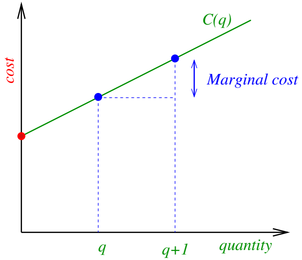


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**Definition**

If $C(q)$ is a linear cost function,

- Fixed cost is represented by the vertical intercept.
- Marginal cost is represented by the slope.

# The Revenue Function

**Definition**

The **revenue function**, $R(q)$, gives the total revenue revived by a firm from selling a quantity, $q$, of some good.

- If the good sells for a price $p$ per unit, and the quantity sold is $q$, then

$$\text{Revenue} = \text{Price} \times \text{Quantity} = pq$$

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- The graph of $R(q) = pq$ is a line passing (0,0), and having slope $p$.

# Example

Graph the cost function $C(q) = 6 + 2q$ and the revenue function $R(q) = 4q$. For what values of $q$ does the company make money?

# The Profit Function

**Definition**

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\pi = R - C$$

The **break-even point** for a company is the point where the profit is zero and revenue equals cost.

# Example

| q | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 |
|---|---|---|---|---|---|---|---|
| C(q) | 5000 | 5500 | 6000 | 6500 | 7000 | 7500 | 8000 |
| R(q) | 4000 | 4800 | 5600 | 6400 | 7200 | 8000 | 8800 |

- Estimate the break-even point.

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- Estimate the break-even point.
- Find the company's profit if 900 units are produced.
- What price do you think the company is charing for its product?

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# The Marginal Cost, Marginal Revenue, and Marginal Profit

**Definition**

- The **marginal cost** is the rate of change, or slope, of linear cost function.
- The **marginal revenue** is the rate of change, or slope, of linear revenue function.
- The **marginal profit** is the rate of change, or slope, of linear profit function.

**Problem 1.** Suppose that the radio manufacture has a machine that costs \$15,000 and is sold ten years later for \$5,000. We say the value of the machine depreciates from \$15,000 today to resale value of \$5,000 in ten years. The depreciation formula gives the value, $V(t)$, in dollars, of the machine as a function of the number of years, $t$, since the machine was purchased. We assume that the value of the machine depreciates linearly. Find $V(t)$.

**Problem 2.** Suppose you have a budget of 400 dollars for one month. You would like to buy some textbooks and CDs. The average cost of a book is 50 dollars each and that of a CD is 10 dollars each. Let x denote the number of books you buy and y denote the number of CDs that you buy. Assume that all the money is spent. Find the relation between $x$ and $y$.

- $$\text{Amount spent on books} + \text{Amount spent on CDs} = \$400$$

# A budget constraint

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- $$\text{Amount spent on books} + \text{Amount spent on CDs} = \$400$$

- $$50x + 10y = 400$$

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- $$\text{Amount spent on books} + \text{Amount spent on CDs} = \$400$$
- $$50x + 10y = 400$$
- $$5x + y = 40$$

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- $$\text{Amount spent on books} + \text{Amount spent on CDs} = \$400$$
- $$50x + 10y = 400$$
- $$5x + y = 40$$
- $$y = 40 - 5x$$