Calculus for the Biological Sciences

New functions from old.

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New functions from old.

Composite functions.

Definition

The function $f(g(t))$ is a function of function, or **composite function**, in which there is an **inside function** and an **outside function**.

Examples

- Assume that $f(x) = 2x^2 + 1$, $g(x) = 3x + 5$, and $h(x) = \frac{1}{x+1}$.

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- Find $f(g(x))$ and $g(f(x))$.

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- Find $f(x + 4)$.
- Find $g(x - 2)$.
- Find $f(g(x))$ and $g(f(x))$.
- Find $h(f(x) + g(2x))$.

Examples

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- Find $f(g(x))$
- Find $g(f(x))$
- Find $f\left((g(x))^2\right)$

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- $f(x) = ln(3t + 1)$
- $y = 5(2x^2 + 4)^2$
- $P = (3x + 1)e^{9x+3}$

Examples

x	0	1	2	3	4	5
f(x)	10	6	3	4	7	11
g(x)	2	3	5	8	12	15

- Find $f(g(0))$.

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- Find $f(g(0))$.
- Find $f(g(1))$.

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- Find $f(g(0))$.
- Find $f(g(1))$.
- Find $f(g(2))$.
- Find $g(f(2))$.
- Find $g(f(g(0)))$.

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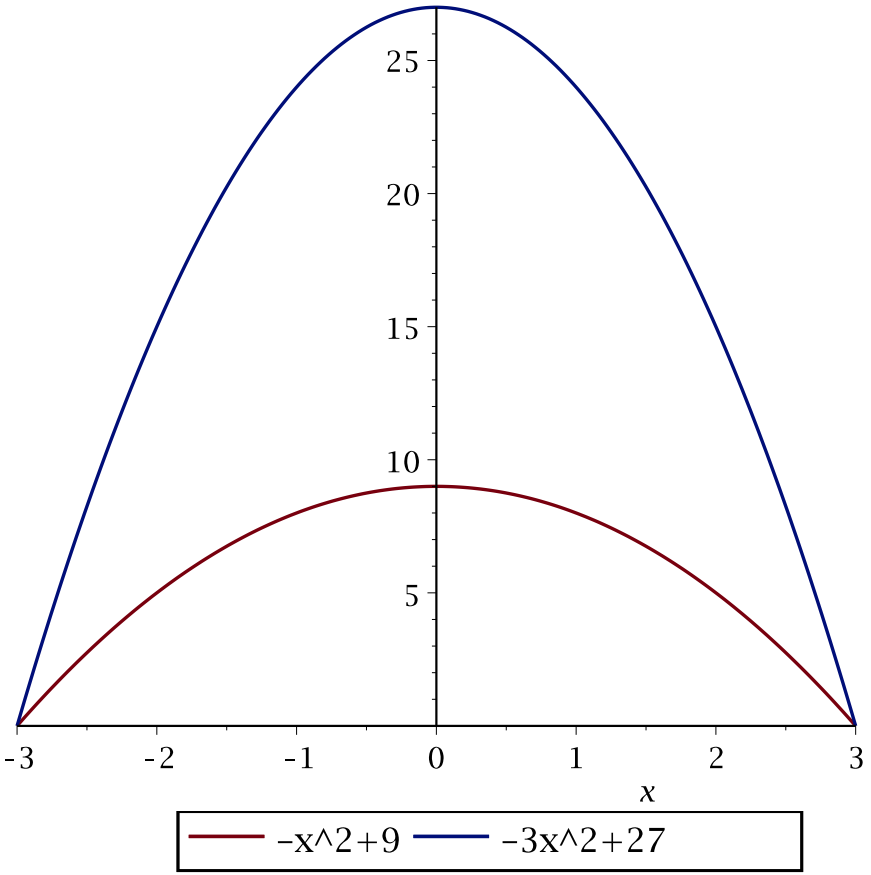
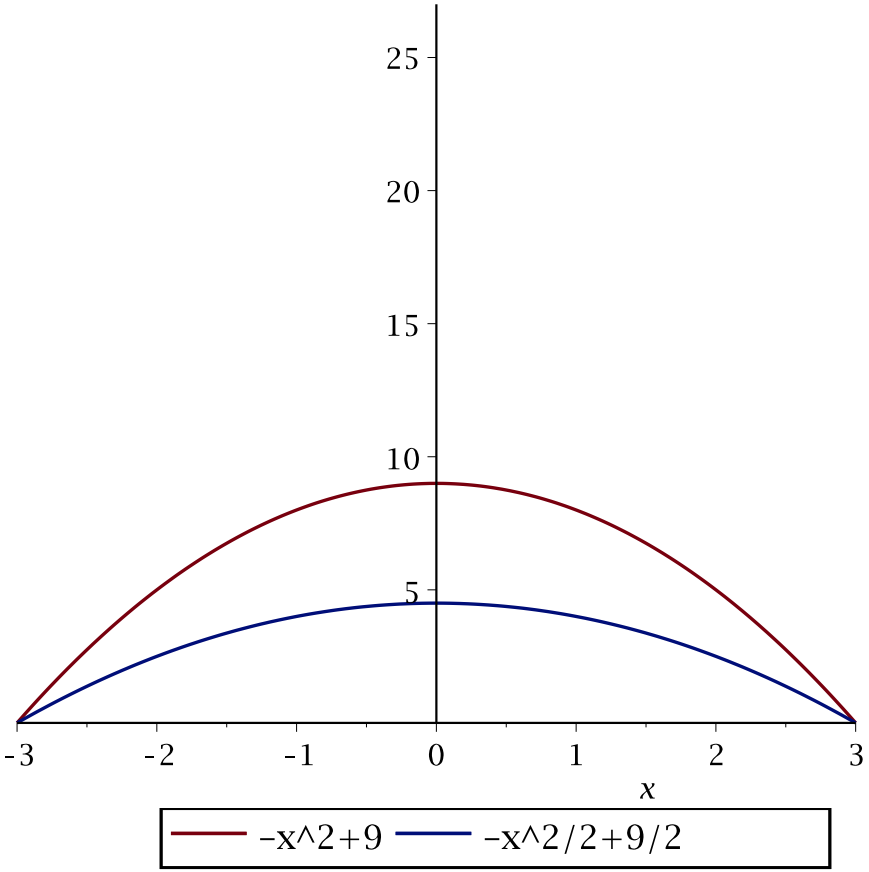
- Multiplying a function by a constant c stretches the graph vertically (if $c > 1$) or shrinks the graph vertically (if $0 < c < 1$).
- A negative sign (if $c < 0$) reflects the graph about x-axis in addition to shrinking or stretching.

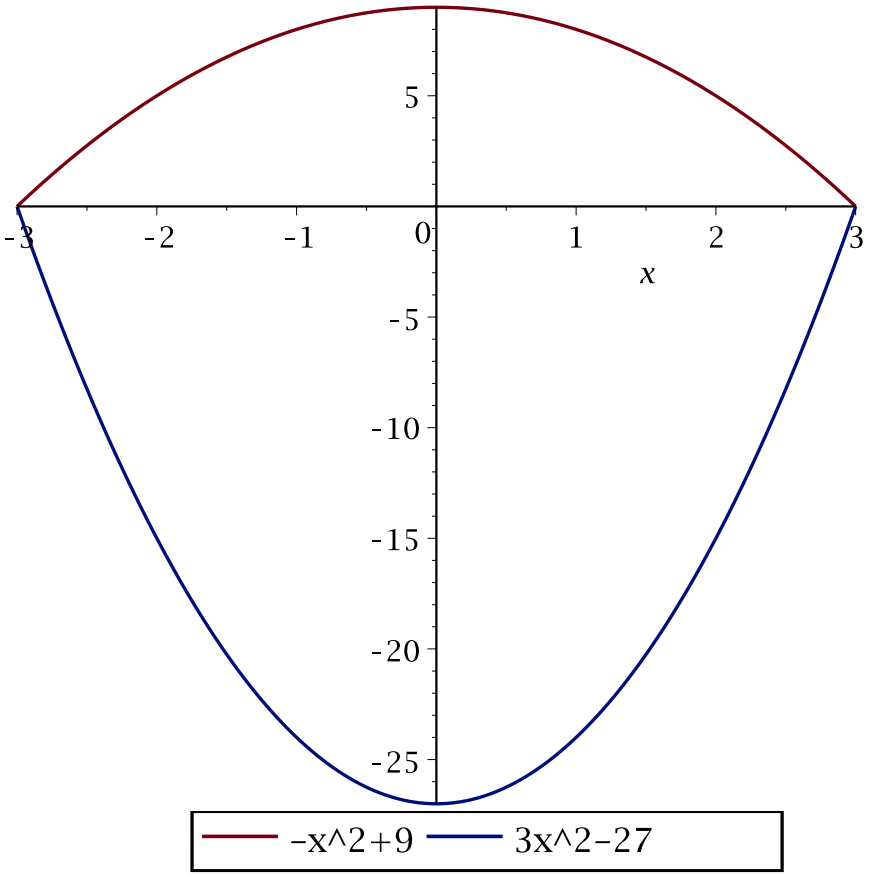
$c > 1$



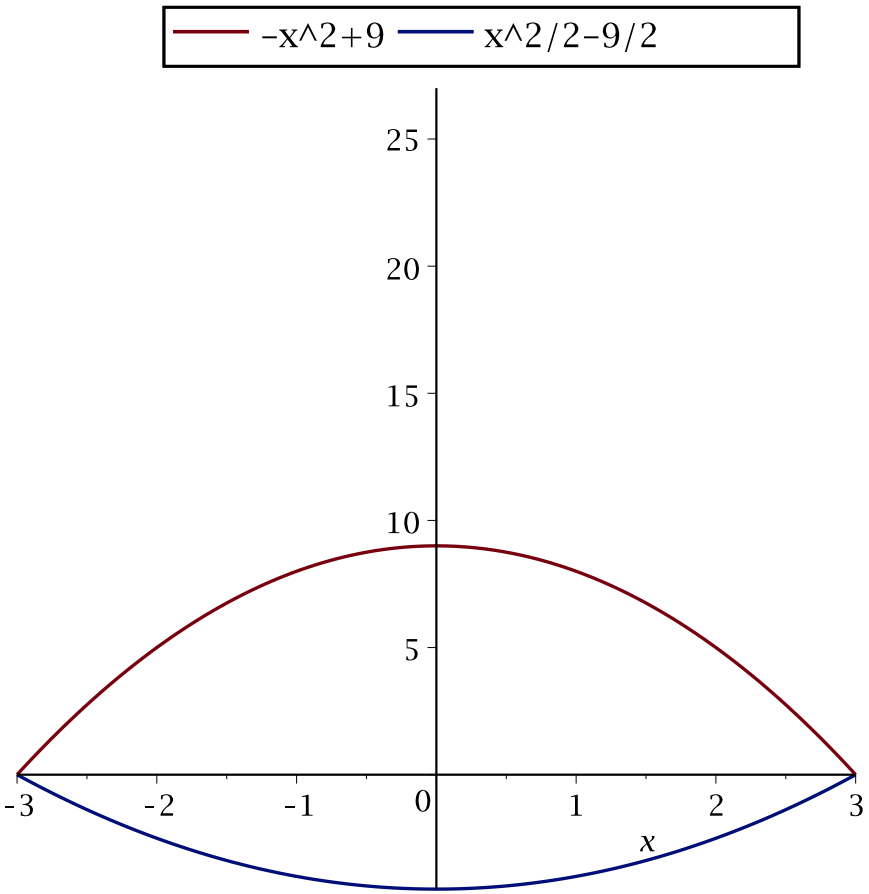
$0 < c < 1$



$c < -1$



$-1 < c < 0$



Shifted graph

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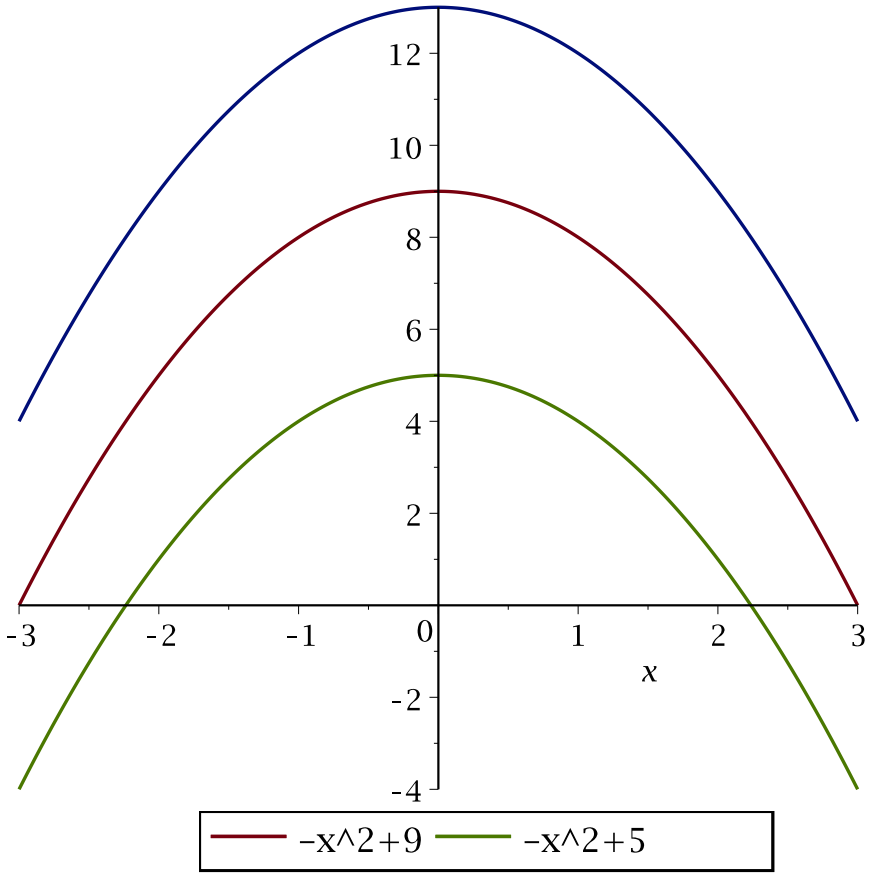
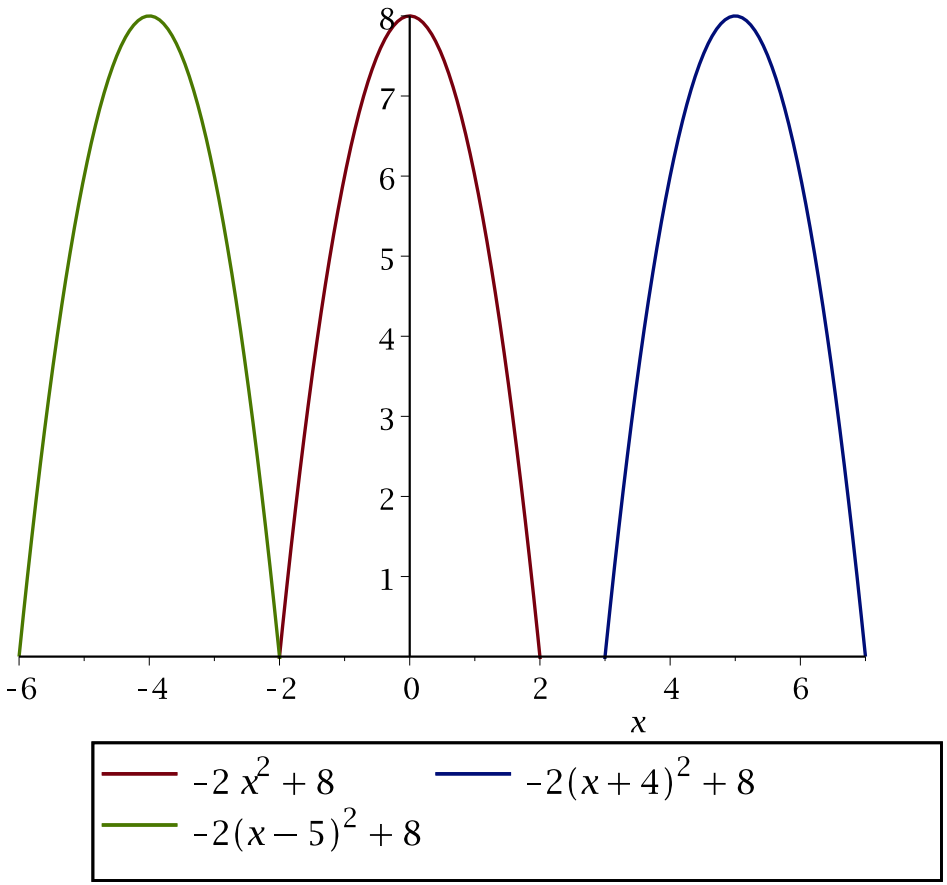
- The graph of $y = f(x - k)$ is the graph of $y = f(x)$ moved to the right k units (to the left if if k is negative).

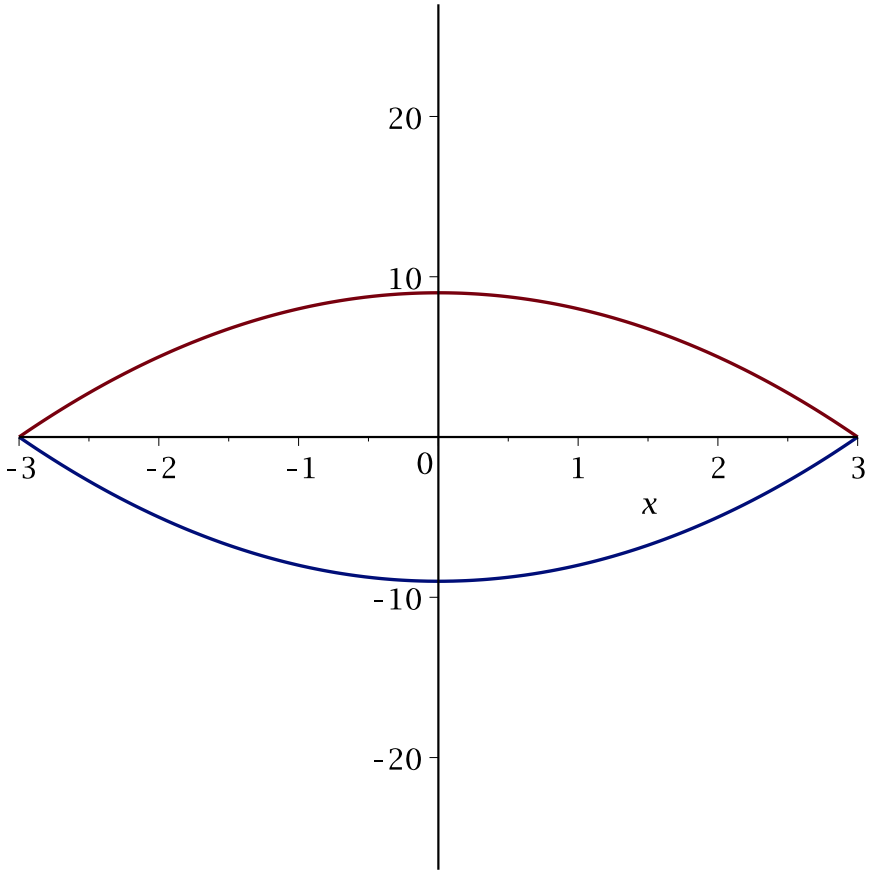
Vertically shift



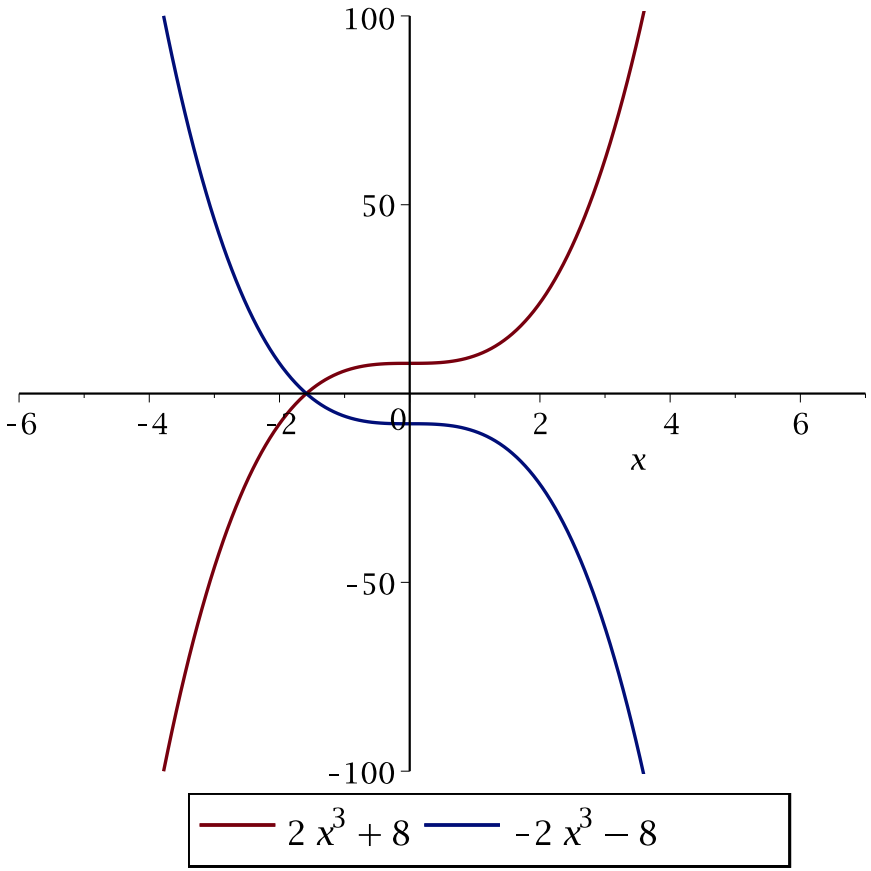
Horizontally shift



Reflection



Refection



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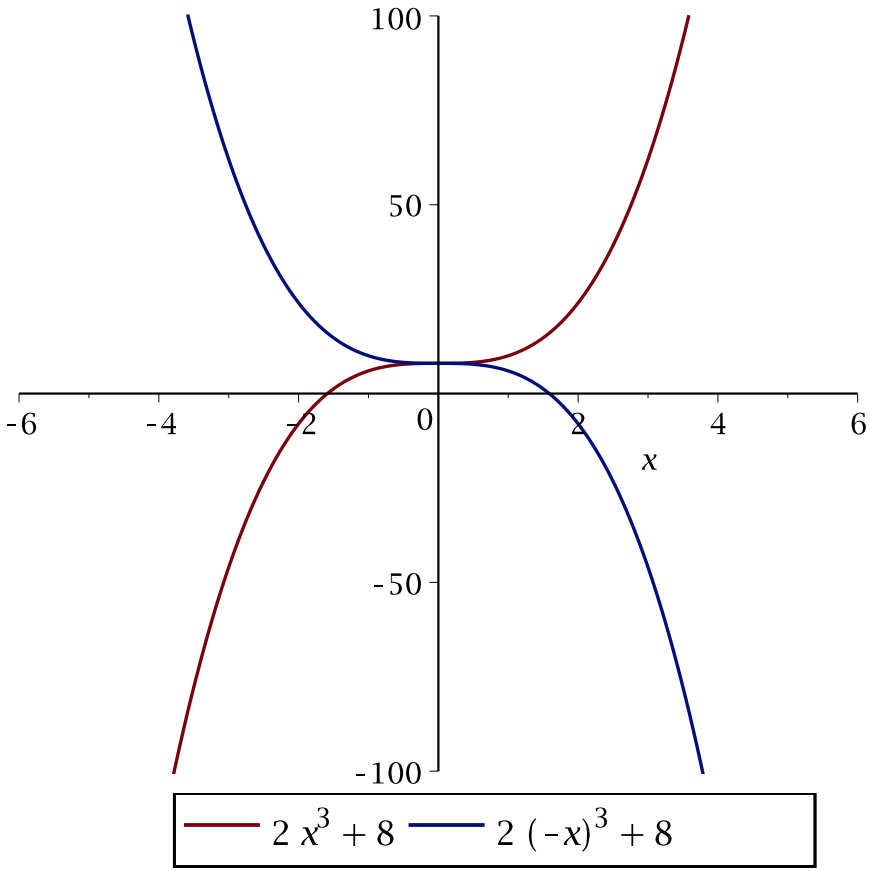

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- The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about the y-axis.