Calculus for the Biological Sciences

Proportionality and power functions

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Fall 2020

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Proportionality

A common functional relationship occurs when one quantity is **proportional** to another. If peaches are \$0.99 a pound, then we say the price you pay, p dollars, is proportional to the weight you buy, q pounds, because

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- This k is called the **constant of proportionality**.
- We also say that y is **inversely proportional** to x if y is proportional to $\frac{1}{x}$.

Examples

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- Human with a body mass of 70 kg has heart mass of 0.42 kg.
- Estimate the heart mass of horse with a body mass of 650 kg.

Examples

The period of a pendulum, T, is the amount of time required for the pendulum to make one complete swing. For small string, the period, T, is approximately proportional to the square root of l, the pendulum length. So

$$T = k\sqrt{l}$$

.

Examples

An object's weight, w, is inversely proportional to the square of its distance, r, from the earth's center. So

$$w = \frac{k}{r^2}$$

Power function

Definition

We say $Q(x)$ is a power function of x if $Q(x)$ is proportional to a constant power of x.

$$Q(x) = k \cdot x^p$$

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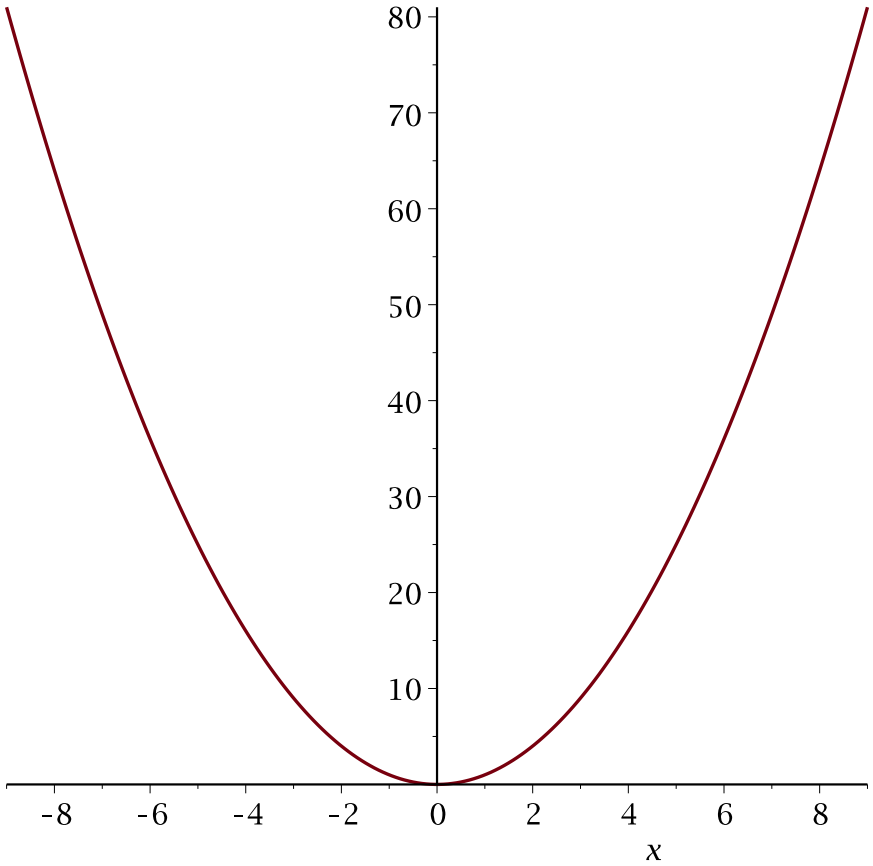
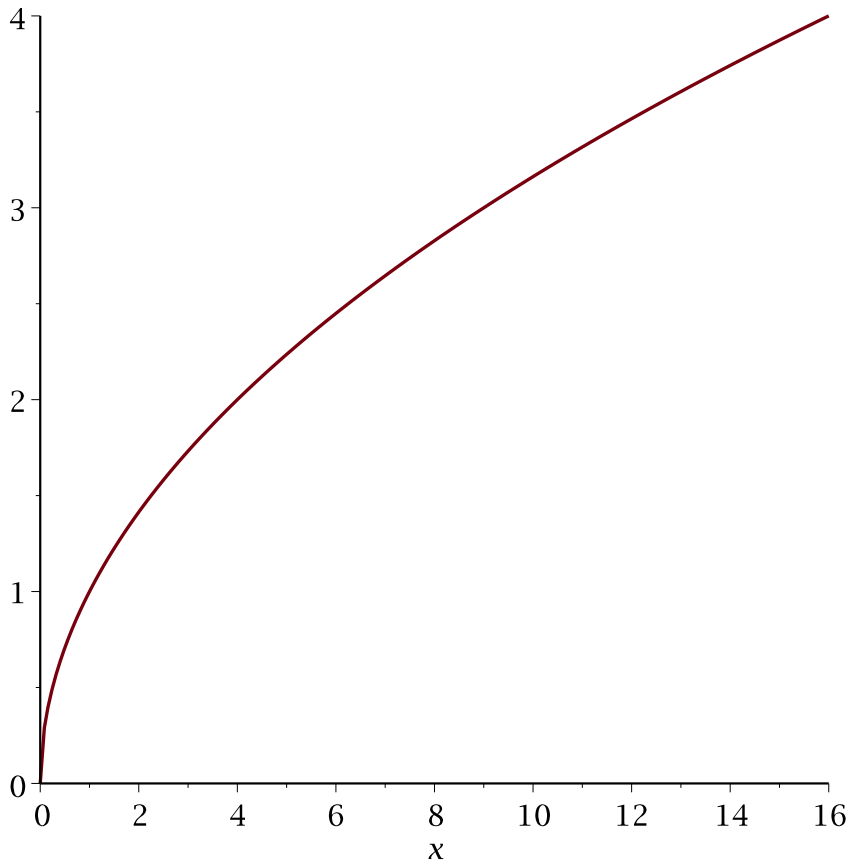
- $y = (4x^3)^4$.

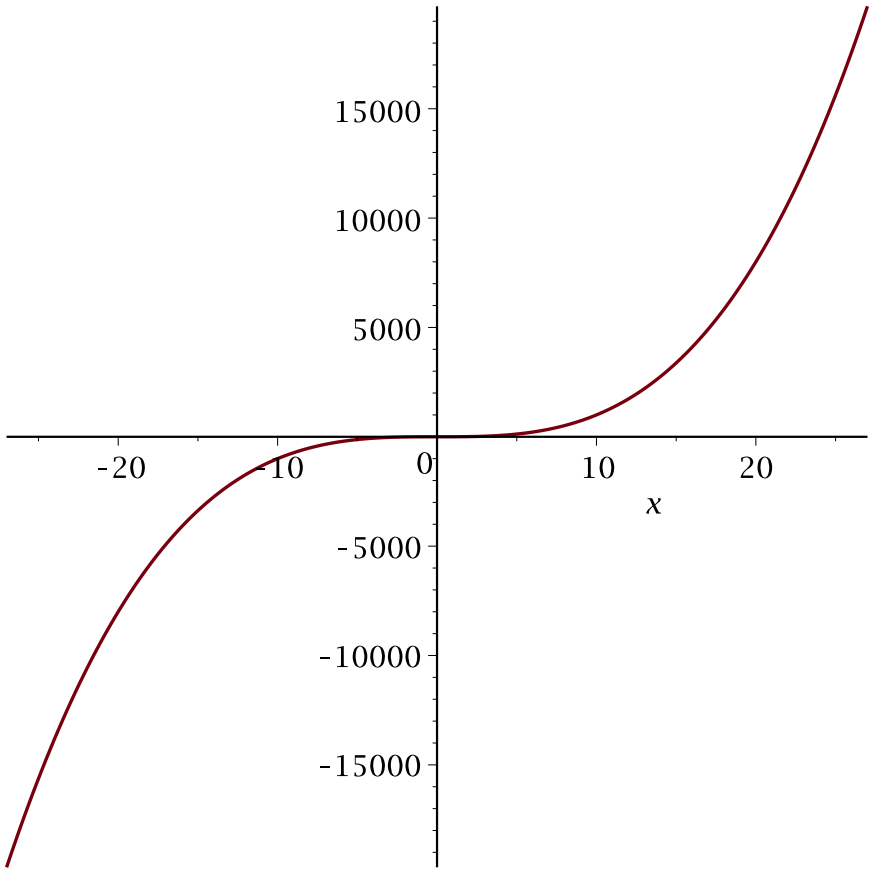
Graph of power functions: $y = x^2$



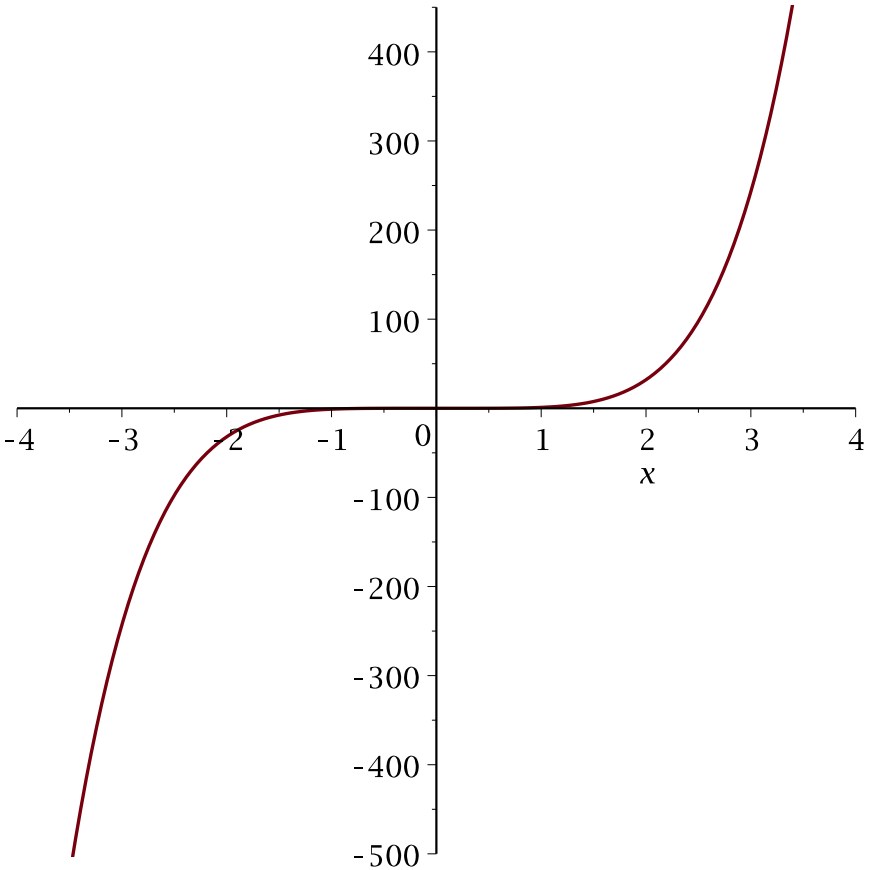
Graph of power functions: $y = \sqrt{x}$



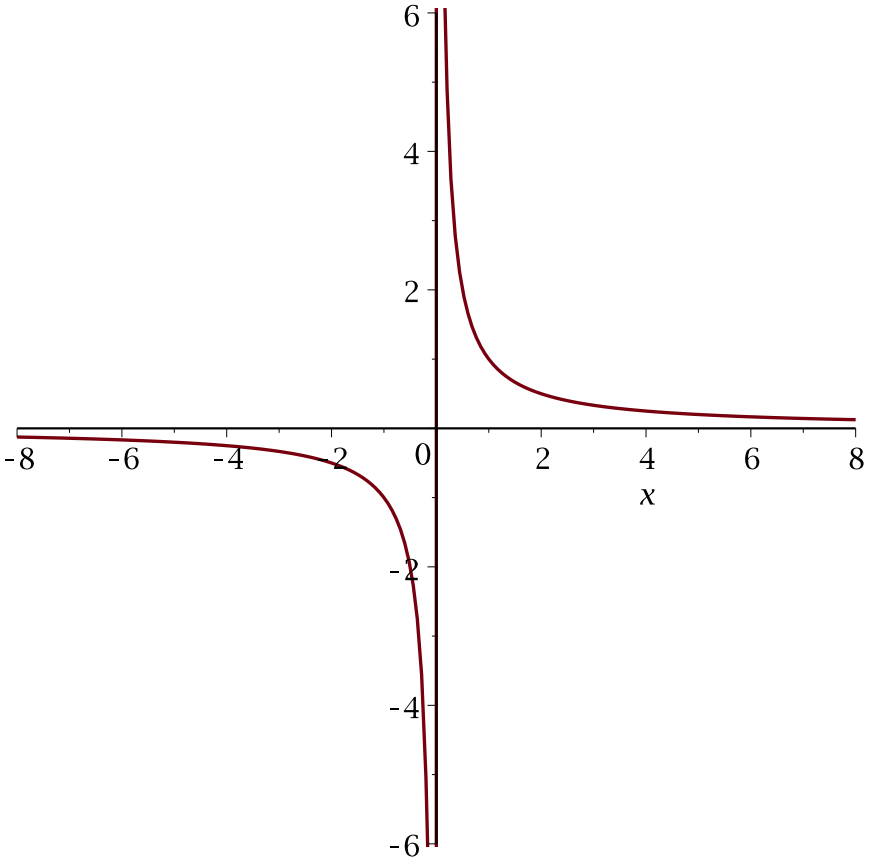
Graph of power functions: $y = x^3$



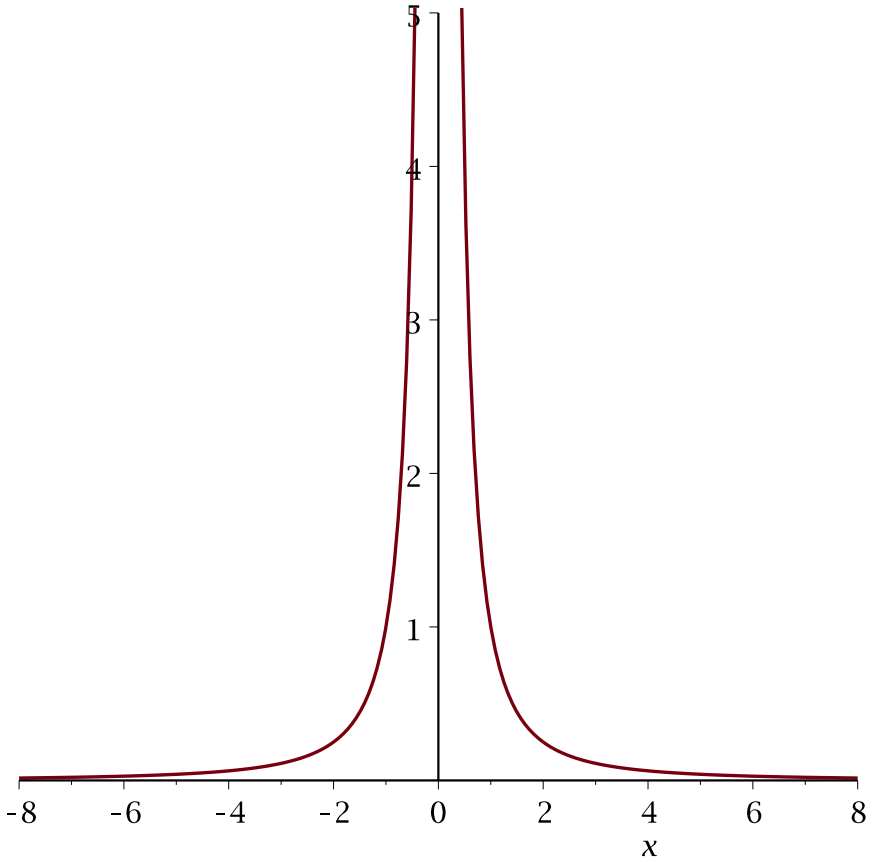
Graph of power functions: $y = x^5$



Graph of power functions: $y = \frac{1}{x}$



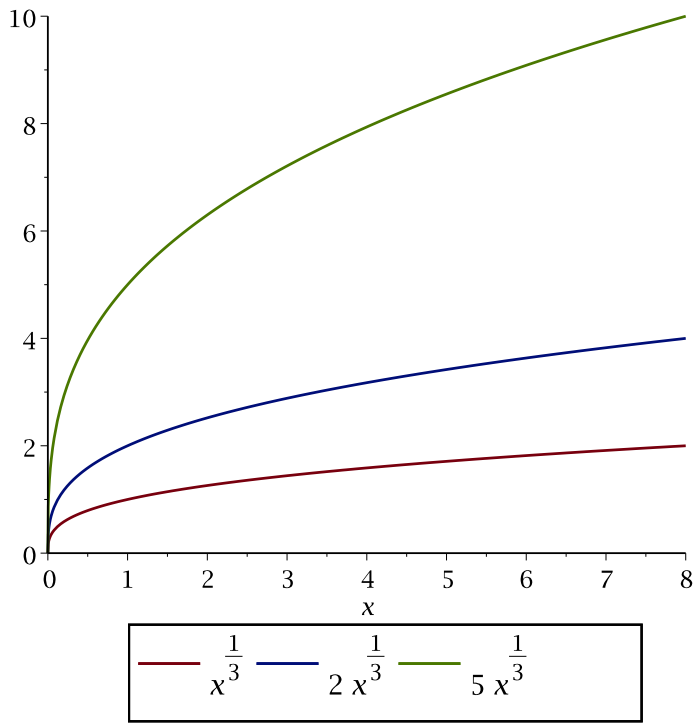
Graph of power functions: $y = \frac{1}{x^2}$



Examples

If N is the average number of species found on an island and A is the area of the island, observations have shown that N is approximately proportional to the cube root of A. So

$$N = k\sqrt[3]{A}.$$



Quadratic functions and Polynomials

Definition

- Sums of power functions with nonnegative integer exponents are called **polynomials**, which are function of the form

$$y = p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0.$$

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- If $n = 2$, the polynomial is called **quadratic function** and has the form $ax^2 + bx + c$ with $a \neq 0$.

Parabola