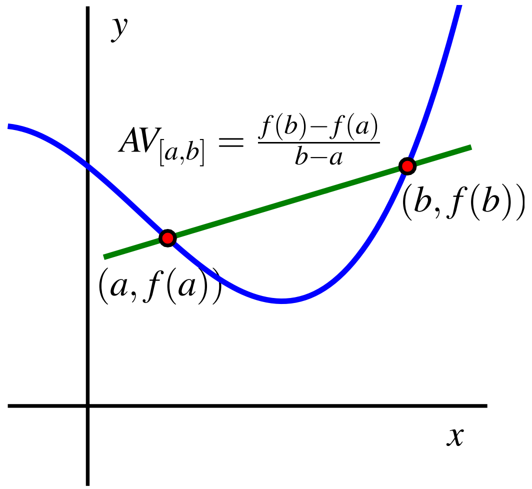
The Average Rate of Change of a Function

•For a function f defined on an interval [a,b], the average rate of change of f on [a,b] is the quantity

$$AV[a,b]=[f(b)-f(a)]/[b-a].$$



- **Example:** Find the average rate of change of y=6x-1 between x= 1 and x=4

- Average rate of change = change in y/ change in x

= [y2-y1]/ [x2-x1]

= [23-5]/[4-1]

= 18/3

=6

Note for a linear function the average rate of change is equal to the slope m=6.

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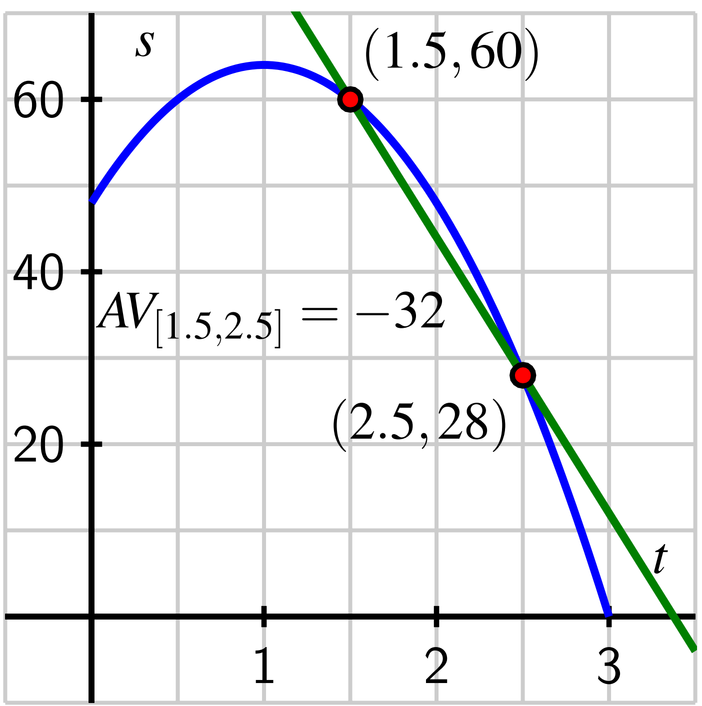
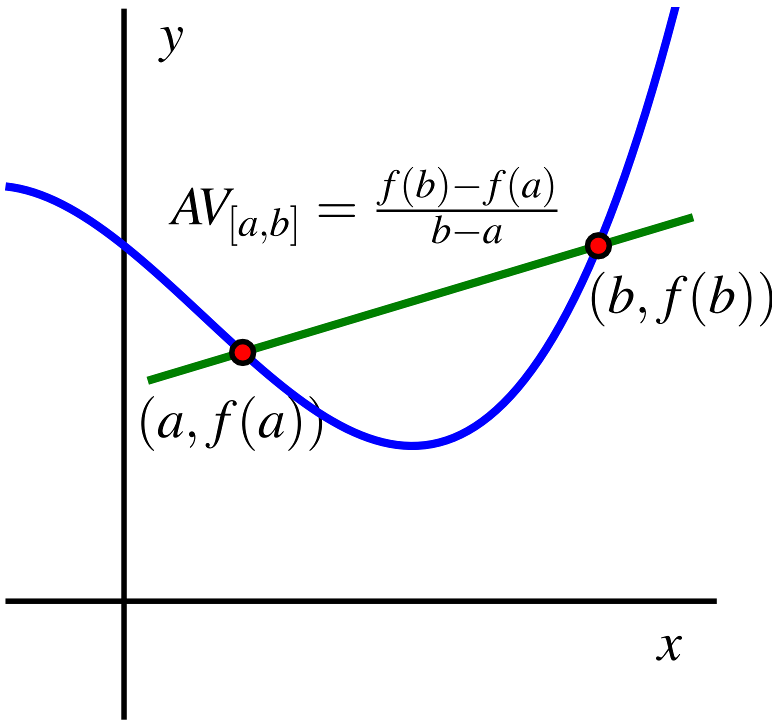




Interpretation

- For a function f defined on an interval [a,b], the average rate of change of f on [a,b] is the quantity

$$AV[a,b]=[f(b)-f(a)]/[b-a].$$

- The value of $AV[a,b]=[f(b)-f(a)]/[b-a]$ tells us how much the function rises or falls, on average

- The value of $AV[a,b]$ is also the slope of the line that passes through the points $(a,f(a))$ and $(b,f(b))$ on the graph of f

Distance, Average velocity

- **Average velocity= change in distance/change in time unit: [m/s]**

V=[Y2-Y1]/[t2-t1]

- =Average rate of change of the distance with respect to time
- Example:

One way you use average rate of change is **Average Velocity**

$$\text{Average Velocity} = \frac{\text{change in Distance}}{\text{change in Time}} = \frac{\Delta D}{\Delta t}$$

<u>Try It Yourself #2</u> Use the table below to calculate the car's average velocity over each interval.

Time (hours)	0	2	4	6	8
Distance (miles)	0	100	180	240	340

1. Between $t=0$ and $t=8$

2. On the interval $[2,4]$

Solution

1) V1= 340/8 miles/hour

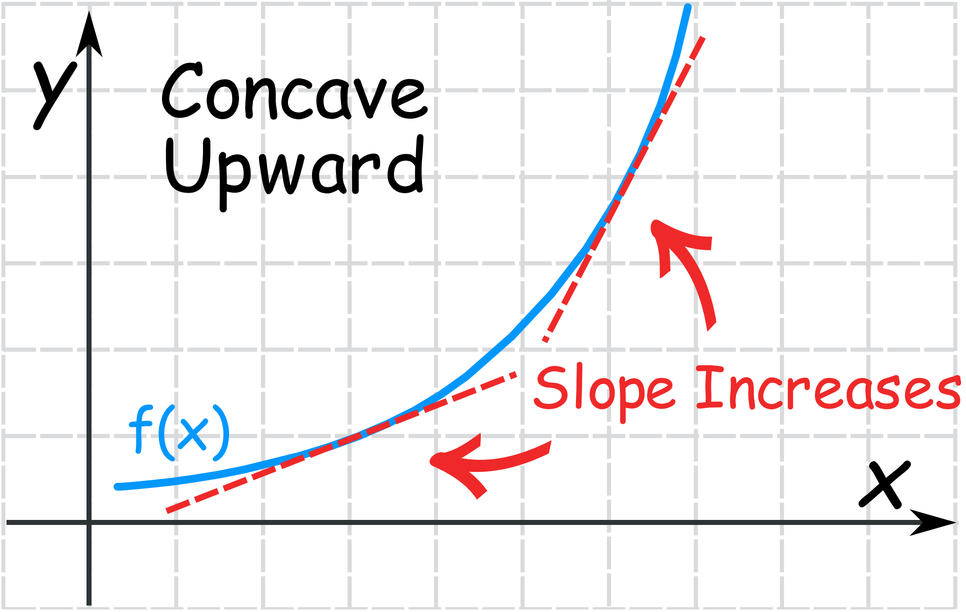
2) V2= [180-100]/[4-2]=80/2=40 miles/h

Increasing or decreasing function

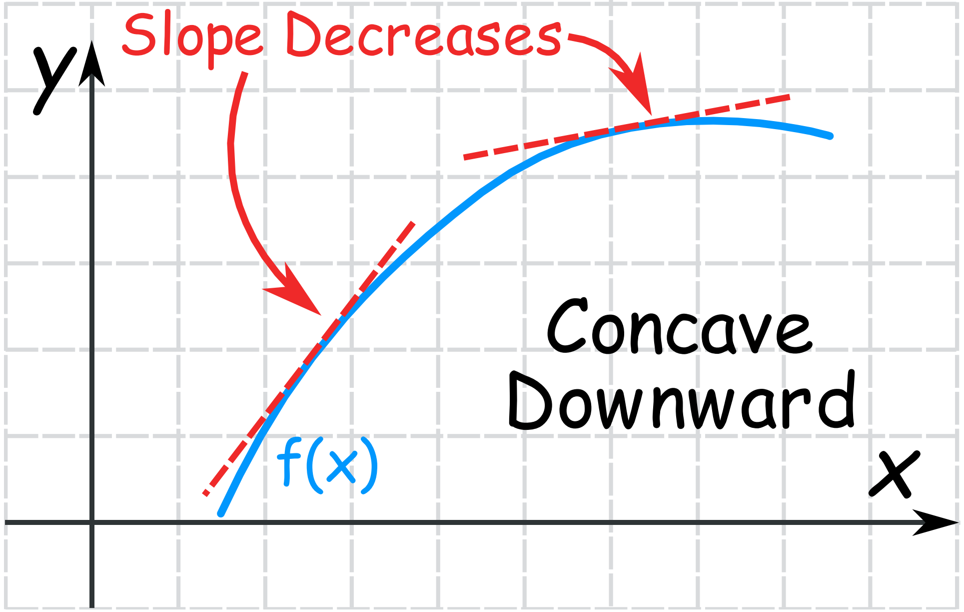
- Let f be a function defined on an interval (a,b) (that is, on the set of all x for which ($a<x<b$). We say that f is ***increasing on*** (a,b) provided that the function is always rising as we move from left to right. That is, for any x and y in (a,b), if $x<y$, then $f(x)<f(y)$.
- Similarly, we say that ff is ***decreasing on*** (a,b) provided that the function is always falling as we move from left to right. That is, for any x and y in (a,b), if $x<y$, then $f(x)>f(y)$.

Concavity

Concave upward is when the slope increases:

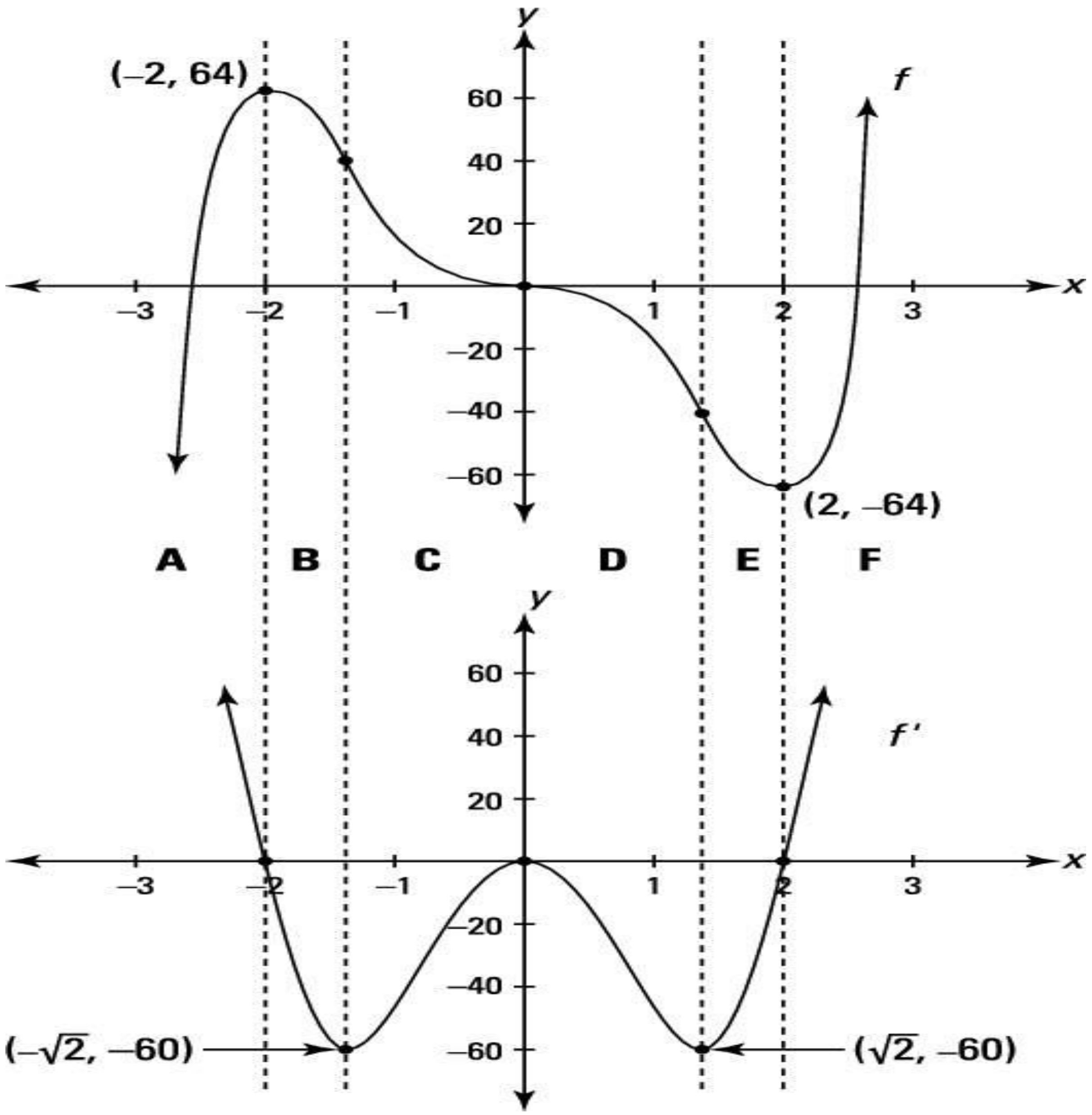


Concave downward is when the slope decreases:



Example:

1- Find the domain D
and the range R of f
2- Find critical points of f
3- Find the x-intercepts of f
3- Find the y-intercept of f
4-Find the interval where
a- f is increasing
b- where f is concave up



Relative change

- When a quantity P changes from P0 to P1, We define:

  Relative change in P= change in P/P0 = [P1-P0]/P0

- It is a number without a unit often expressed as a percentage

Example:

- If the population increases by 100 people. Find the relative change if the initial population P0 is equal to 1559

- Relative change = change in population/ initial population

  = 100/1559 =0.64

Which means that the population has increased by 64%