Calculus for the Biological Sciences

What is a function?

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- The set of resulting output numbers is called the range of the function.

Function Examples

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Average low F	21.2	23.2	32.1	42.6	52.5	61.4	65.2	63.4
Record low F	-21	-20	-2	17	29	36	46	41
Record low C	-29.4	-28.9	-18.9	-8.3	-1.7	2.2	7.8	5

Table : Climate date for Bloomington IN from January to August

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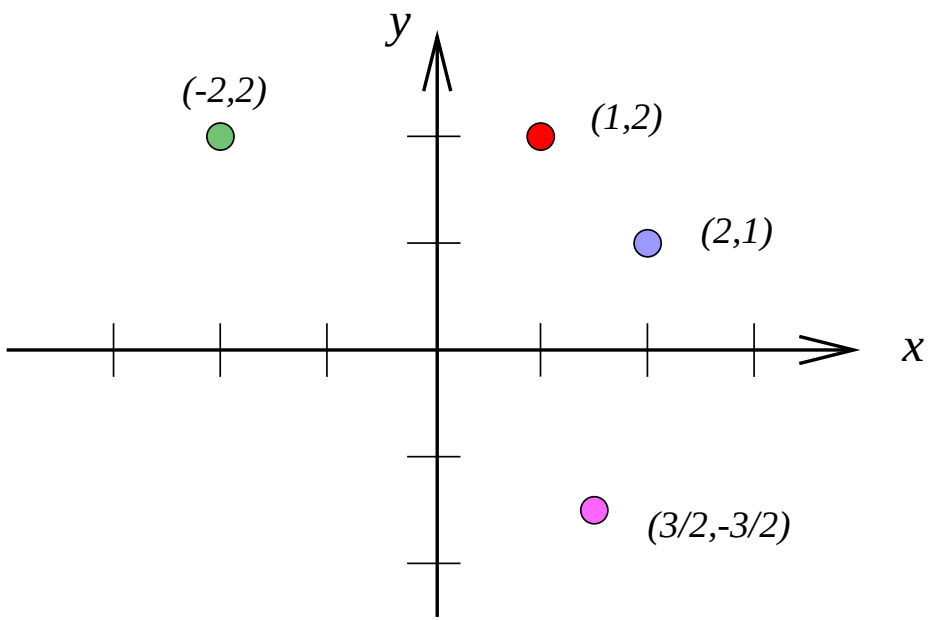
- Temperature conversion formula: $(F - 32) \times 5/9 = C$.

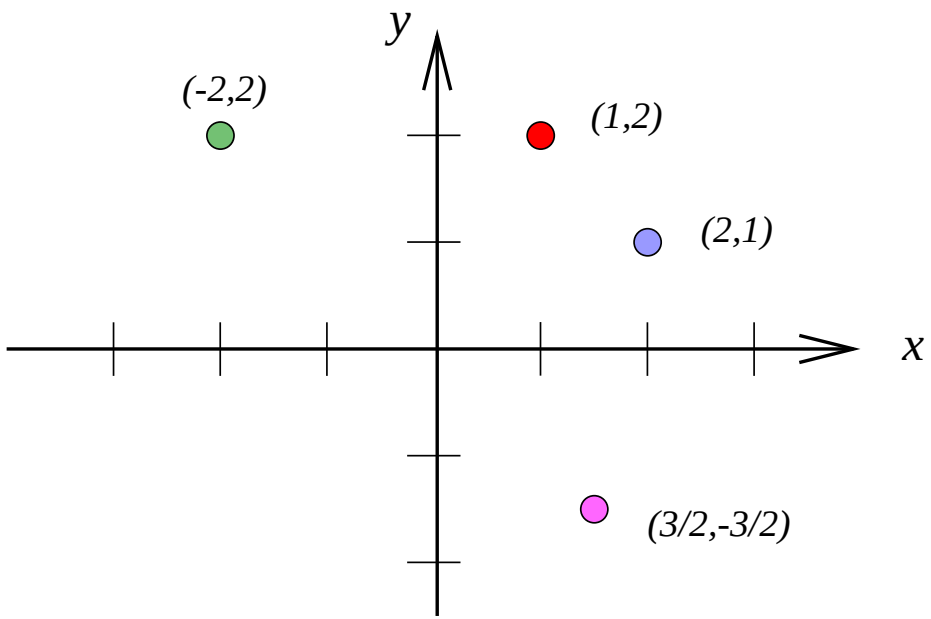
Cartesian Coordinate system

- Associate pairs of numbers with points on the plane.

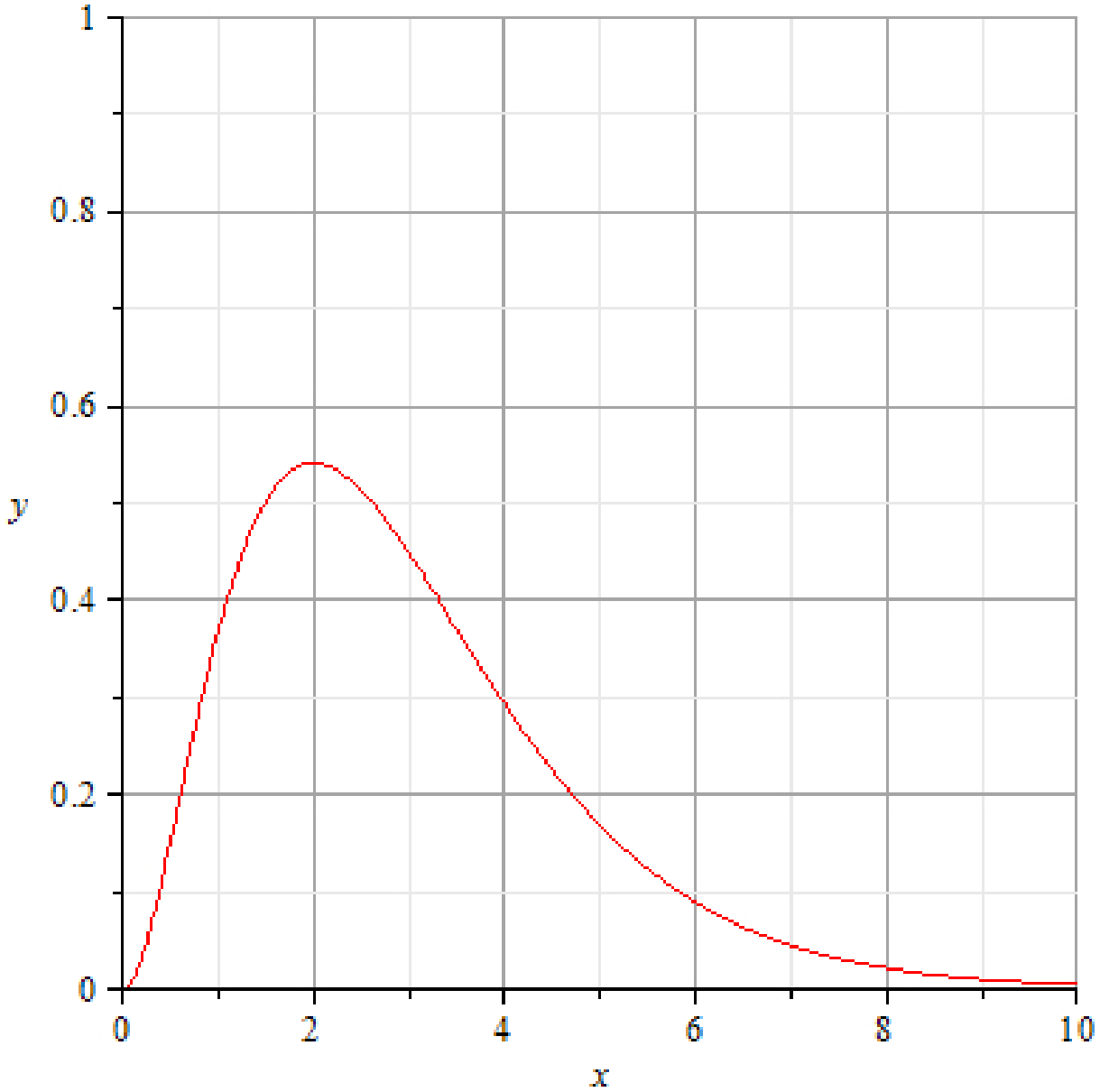


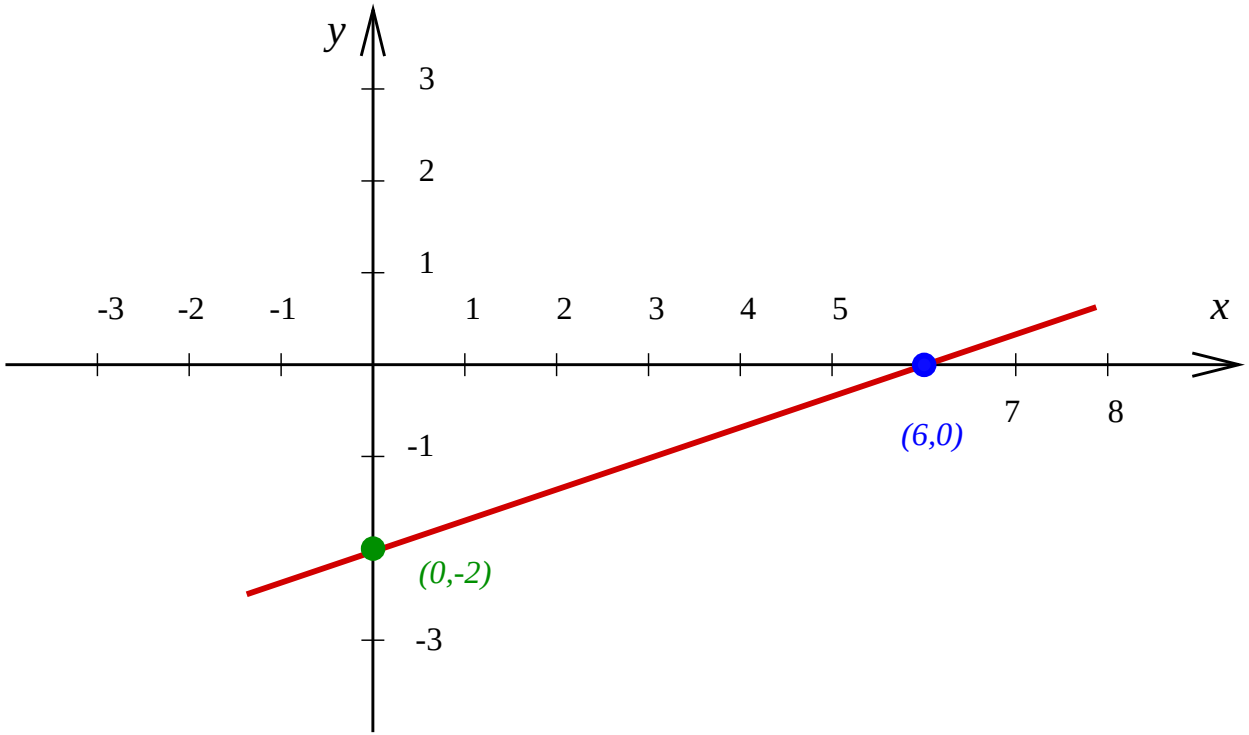
Cartesian Coordinate system

- Associate pairs of numbers with points on the plane.
- If P is the label of point (x, y), then the numbers x and y are the coordinates of point P.



The rule of four: Tables, Graphs, Formulas, and Words




y
x
3
2
1
-3
-2
-1
1
2
3
4
5
7
8
-1
-3
(6,0)
(0,-2)

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- An instructor gives weekly quizzes in her mathematics course, and she seeks to find the relationship between the time required to prepare and grade a quiz and the number of students in the class. Her experience is that the average time required to prepare a quiz is 30 minutes, and the average time to grade each paper is 2 minutes.

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- $$y = 30 + 2x$$

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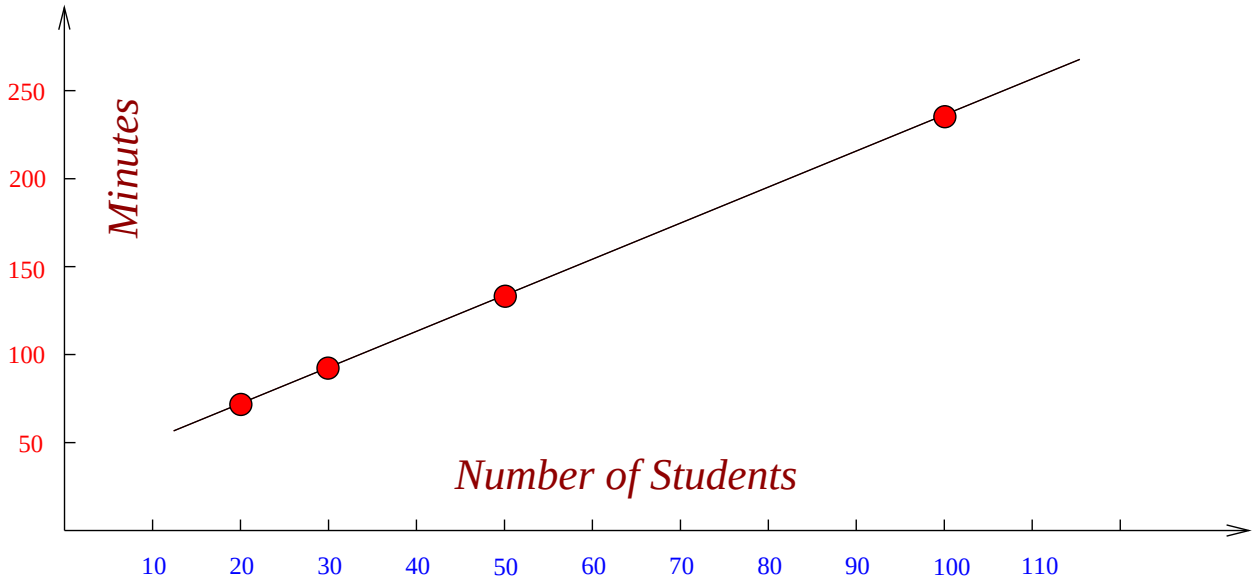

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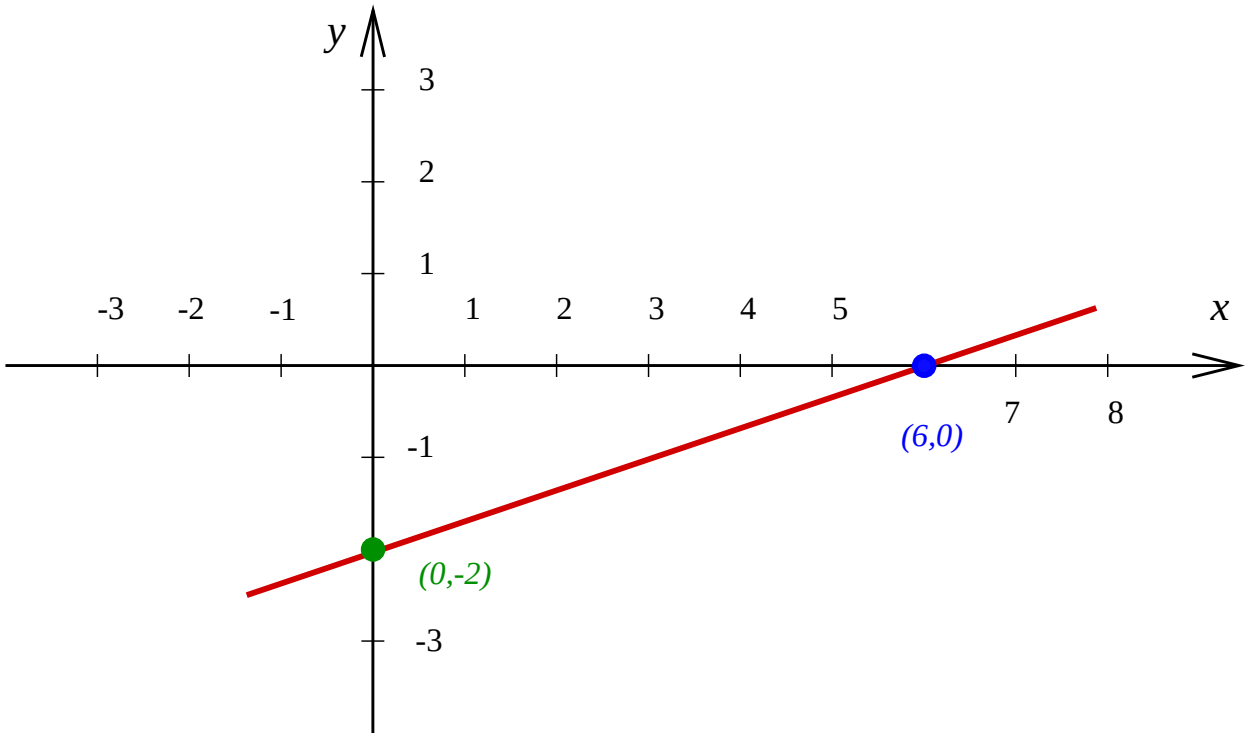
- Horizontal intercepts are also called the **zeros** of the function.

Function notation and intercepts

Find the intercepts of the function $y = x/3 - 2$.



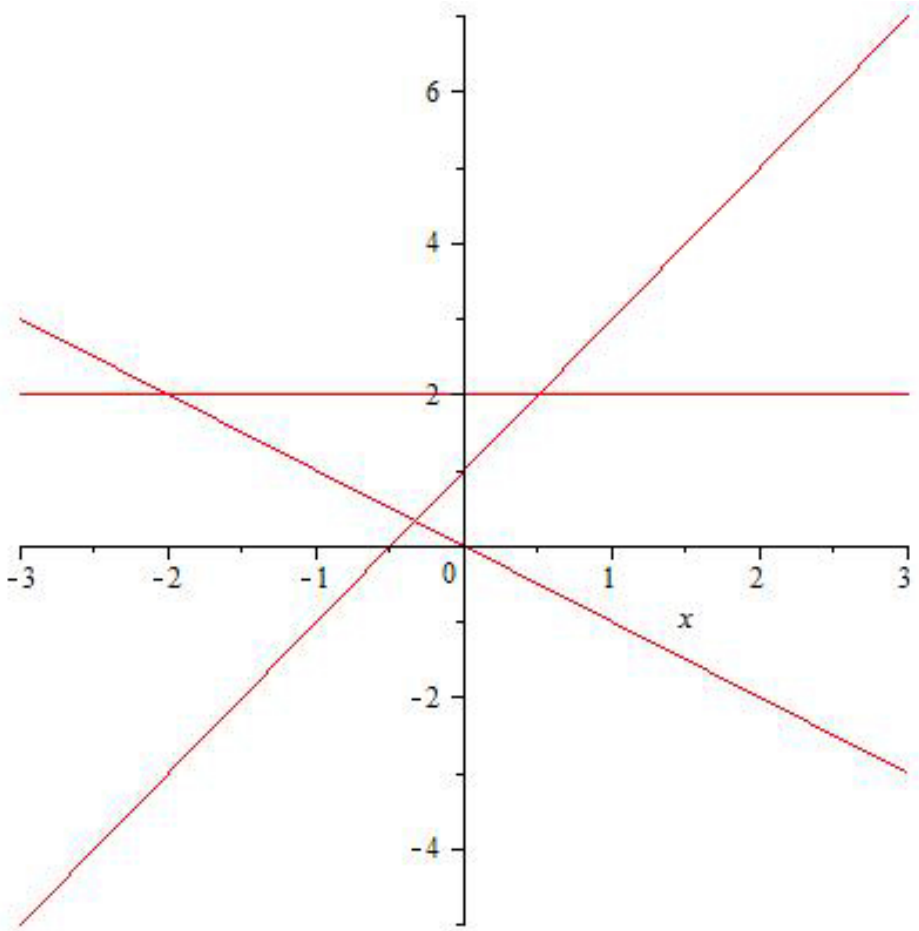
Increasing and decreasing functions

Definition

A function f is **increasing** if the value of $f(x)$ increases as x increases.

A function f is **decreasing** if the value of $f(x)$ decreases as x increases.

Increasing and decreasing functions



Lines

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- If (x_1, y_1) and (x_2, y_2) are two different points on this line, i.e. $Ax_1 + By_1 = C$ and $Ax_2 + By_2 = C$, then the line is referred as the line through (x_1, y_1) and (x_2, y_2).

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- The equation $Ax + By = C$ is called the **general equation** of the line.

Slope

- The **slope** of a line is the ratio of the difference of the y coordinates to the difference of the x coordinates of any two distinct points on the line, whenever the latter difference is not zero.

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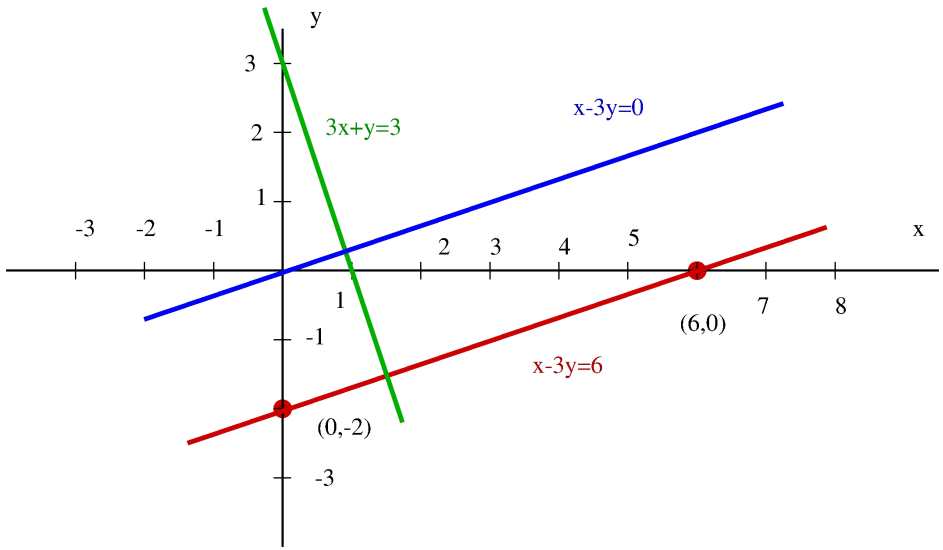
- The slope m of a line does **not** depend on the choice of the points (x_1, y_1) and (x_2, y_2).

Parallel orthogonal lines

- Two lines a parallel if: they have the same slope or if the slope is undefined for both.



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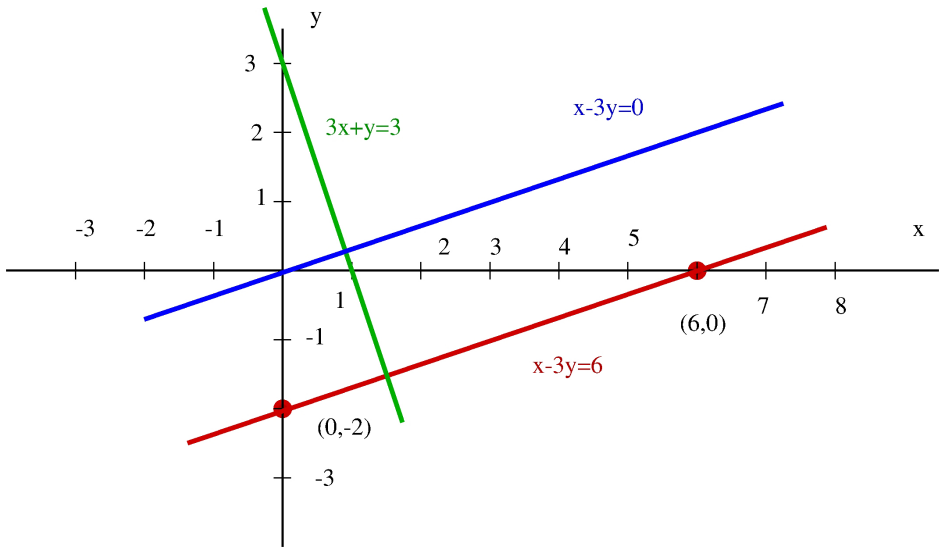
- Two lines are orthogonal if: the slope is undefined for one of them, and the other one has slope 0; or the product of two slopes is -1.



Linear Functions.

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- A linear function has the form

$$y = f(x) = b + mx$$

Recognizing Data from a Linear Function: Values of x and y in a table could come from a linear function $y = b + mx$ if differences in y-values are constant for equal differences in x-values.

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- Its graph is a line such that:
  - m is the slope, or rate of change of y with respect to x.
  - b is the vertical intercept.

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Point-slope form of a line

- Find the equation of the line which has slope 4 and which passes through the point (2,3).

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- $$y - y_0 = m(x - x_0)$$