Math 3100

## Section 1.1: Systems of Linear Equations

**Elementary Row Operations: On a Matrix** If any sequence of the following operations are performed on a matrix, the resulting matrix is **row equivalent**.

- i Interchange any two rows (row swap).
- ii Multiply a row by any nonzero constant (scaling).
- iii Replace a row with the sum of itself and a multiple of another row (**replacement**).

**Theorem:** If the augmented matrices of two linear systems are row equivalent, then the systems have the same solution set. (i.e. The systems are equivalent!)

## A key here is structure!

Consider the following augmented matrix. Determine if the associated system is consistent or inconsistent. If it is consistent, determine the solution set.

(a) 
$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{System} |X_1 + 0Y_2 + 0X_3 = 3$$
  
$$\xrightarrow{OX_1 + 1X_2 + 0X_3 = 1}$$
  
$$\xrightarrow{OX_1 + 0Y_2 + 1X_3 = -2}$$

The system is consistent with solution  $X_1 = 3$   $X_2 = 1$  $X_3 = -2$ 

(b) 
$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$
  $\rightarrow$   $\begin{bmatrix} 1x_1 + 2x_2 + 0x_3 & z & 3 \\ 0x_1 + 1x_2 & -1x_3 & z & 4 \\ 0x_1 + 0x_2 + 0x_3 & z & 3 \end{bmatrix}$ 

(c) 
$$\begin{bmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 \times_1 & t & 0 \times_2 & -2 \times_3 & = 5 \\ 0 \times_1 & t & 1 \times_2 & t & 1 \times_3 & = 4 \\ 0 \times_1 & t & 0 \times_2 & t & 0 \times_3 & = 0 \end{bmatrix}$$
The last equation  $0 = 0$  is always fire.  
The first two equations say
$$X_1 = 5 + 2 \times 3$$
$$X_2 = 4 - X_3$$
$$X_3 \quad (m = be any real number)$$

The system has infinitely many solution

we can express the solution set as  $\left\{ \left( \chi_{1}, \chi_{2}, \chi_{3} \right) \middle| \chi_{1} = S + 2\chi_{3}, \chi_{2} = 4 - \chi_{3}, \chi_{3} \text{ is seal} \right\}$ 

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