Section 1.1: Systems of Linear Equations

We begin with a linear (*algebraic*) equation in *n* variables $x_1, x_2, ..., x_n$ for some positive integer *n*.

A linear equation can be written in the form

$$a_1x_1+a_2x_2+\cdots+a_nx_n=b.$$

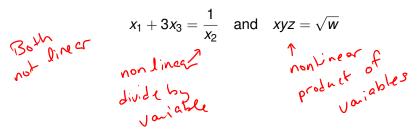
The numbers a_1, \ldots, a_n are called the *coefficients*. These numbers and the right side *b* are real (or complex) constants that are **known**.

Questions

- Is there a set of numbers x₁,..., x₄ that satisfy all of the equations?
 Existence question
- If there is a set of numbers, is it the only one?
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Examples of Equations that are or are not Linear

Both
$$2x_1 = 4x_2 - 3x_3 + 5$$
 and $12 - \sqrt{3}(x + y) = 0$
Linear $5x_1 - 4x_2 + 3x_3 = 5$ $-5x_1 - \sqrt{3}y_2 = -12$



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A *Linear System* is a collection of linear equations in the same variables

$$2x_1 + x_2 - 3x_3 + x_4 = -3$$

-x₁ + 3x₂ + 4x₃ - 2x₄ = 8

$$x + 2y + 3z = 4$$

$$3x + 0y + 12z = 0$$

$$2x + 2y - 5z = -6$$

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Some terms

- ► A solution is a list of numbers (s₁, s₂,..., s_n) that reduce each equation in the system to a true statement upon substitution.
- A solutions set is the set of all possible solutions of a linear system.
- Two systems are called equivalent if they have the same solution set.

An Example

(a) Show that (1,3) is a solution.

Set x=1 and y=3. We have to show that both equations reduce to identities equation 1: $2(1) - (3) \stackrel{?}{=} -1$ 2 - 3 = -1-1 = -1 equation 2: -4(1) + 2(3) = 2-4 + 6 = 22 = 2

(1,3) is a solution.

An Example Continued

$$2x - y = -1$$

 $-4x + 2y = 2$

(b) Note that $\{(x, y)|y = 2x + 1\}$ is the solution set.

Let's show both reduct to identities:
equation 1
$$2x - (2x+1) \stackrel{?}{=} -|$$

 $2x - 2x - 1 = -1$
 $-1 = -1$
equation 2 $-4x + 2(2x+1) \stackrel{?}{=} 2$
 $-4x + 4x + 2 = 2$

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From equation 1: 2x-y = -1add y and 1 to both sides 2x + 1 = y

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