

Subgroups of the additive group of integers exercises for points

In class we talked about the subgroups of the abelian group of integers $\mathbb{Z}$.

(1) Show that all subgroups of $\mathbb{Z}$ are of form $H_a = \{an \mid n \in \mathbb{Z}\}$ for some $a \geq 0$.

(2) Show that $H_a \subseteq H_b$ exactly when $b \mid a$ (integer division).

(3) For all positive a and b show that the intersection group $H_a \cap H_b$ equals H_m , where $m = \text{lcm}(a, b)$, the least common multiple of a and b .

(4) Give an example of positive a and b such that $H_a \cup H_b$ is not a group, and explain why not.

(5) For all positive a and b show that the group $H_a + H_b$ generated by H_a and H_b equals H_d , where $d = \text{gcd}(a, b)$, the greatest common divisor of a and b .

Handwritten answers, due by the last Friday regular class.