Name: ____________________

Take home homework

MATH 3100.101 Linear Algebra etc

USA 31 March 2022

Due date: 11 April 2022.

Working together is OK. Using matrix algebra calculators is also OK. However, turn your *own* version of work in in *handwritten* form. Give exact answers where possible, no approximations unless explicitly stated. So 1/3 is OK where 0.3333333 may be wrong.

1.

(a) A *permutation matrix* is a square matrix of all 0s except for exactly a single 1 in each row and column. Experiment with permutation matrices and come to a strong guess as to what the inverse of a permutation matrix looks like.

(b) The same quest as above, except that each row and column contains exactly one nonzero element. Experiment with such matrices and come to a strong guess as to what the inverse of such a matrix looks like.

(c) Consider an invertible matrix which is the identity except for one column. The same quest as above for the inverse matrix.

2. Find the 3×3 transition matrix A such that

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

where

$$x_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + y_3 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

are the same vector, but expressed over different bases.

3. (a) Find the cosine of the angle between the vectors

$$\begin{bmatrix} 5 \\ -3 \\ 8 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ 21 \\ -13 \end{bmatrix}$$

(b) Find the orthogonal projection of $\vec{v}$ onto $\vec{u}$, where

$$\vec{v} = \begin{bmatrix} 5 \\ -3 \\ 8 \end{bmatrix} \quad \text{and} \quad \vec{u} = \begin{bmatrix} 2 \\ 21 \\ -13 \end{bmatrix}$$

4.
(a) Use Gram-Schmidt to find an orthonormal basis of the subspace of $\mathbb{R}^4$ generated by the list of three vectors

$$\begin{bmatrix} 2 \\ -1 \\ 4 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 3 \\ -8 \\ 21 \end{bmatrix} \quad \begin{bmatrix} -2 \\ 5 \\ 11 \\ 9 \end{bmatrix}$$

(b) Use Gram-Schmidt to find an orthonormal basis of the subspace of $\mathbb{R}^3$ generated by the list of two vectors

$$\begin{bmatrix} 2 \\ x \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$