Name

Take home homeworkIvanieMATH 4120.101Abstract Algebra 1USA20 November 2022

Due date: 4 December 2022 or so.

Working together is OK. However, turn your *own* version of work in in *hand-written* form. Justify your answers.

Instead of the cube group of 24 matrices of size 3×3 , we only work with the square group of 8 matrices of size 2×2 . Let us call the group G. It is generated by matrices

$$R = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Recall that G is not abelian (commutative). For example, $SR = R^3S$

(a) A list of easy questions about G. Make sure that you answer them all. Find an element of G of order 1. Find four elements of G of order 2. Find an element of G of order 3, or explain why none exists. Find two elements of G of order 4.

Find an element of G of order 8, or explain why none exists.

(b) Group G acts on the plane $X=\mathbb{R}^2$ by the usual matrix multiplication. So for example

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

Find a vector $\vec{x} \in \mathbb{R}^2$ with orbit size $|O_{\vec{x}}| = 1$. Find a vector $\vec{x} \in \mathbb{R}^2$ with orbit size $|O_{\vec{x}}| = 4$. Find a vector $\vec{x} \in \mathbb{R}^2$ with orbit size $|O_{\vec{x}}| = 8$.