Name

Take home homework 3

MATH 3100.101 Linear Algebra etc

USA 19 April 2023

Due date: 3 May 2023.
Working together is OK. Using MATLAB is also OK. However, turn your *own* version of work in in *handwritten* form. Give exact answers where possible, no approximations unless explicitly stated. If you use MATLAB, give the commands typed in for your answer.

1. Are the list of vectors $\mathbf{v}_1 = \begin{bmatrix} 4 \\ -5 \\ 3 \\ -1 \\ 7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 5 \\ 4 \\ 1 \\ 1 \\ 6 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 4 \\ 1 \\ 1 \\ 6 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 6 \\ -9 \\ -1 \\ 7 \\ 0 \end{bmatrix}$ a basis for vector space $\mathbb{R}^5$? If yes, why. If no, why not. We only accept simple correctly formulated answers.

2. Are the list of vectors $\mathbf{v}_1 = \begin{bmatrix} 4 \\ -5 \\ 3 \\ -1 \\ 7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 4 \\ 1 \\ 1 \\ 6 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 0 \\ 4 \\ 1 \\ 1 \\ 6 \end{bmatrix}$, $\mathbf{v}_5 = \begin{bmatrix} 6 \\ -9 \\ -1 \\ 7 \\ 0 \end{bmatrix}$ a basis for vector space $\mathbb{R}^5$? If yes, why. If no, why not. We only accept simple correctly formulated answers.

3. Are the list of vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{v}_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 7 \\ 0 \end{bmatrix}$ a basis for vector space $\mathbb{R}^5$? If yes, why. If no, why not. We only accept simple correctly formulated answers.

4. Are the list of vectors $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 8 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 7 \\ 4 \\ 8 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 9 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 0 \\ -8 \\ 2 \\ 6 \\ 6 \end{bmatrix}$, $\mathbf{v}_5 = \begin{bmatrix} 9 \\ 0 \\ 0 \\ 5 \\ 6 \end{bmatrix}$, $\mathbf{v}_6 = \begin{bmatrix} 7 \\ -5 \\ 3 \\ 7 \\ 2 \end{bmatrix}$ a basis for vector space $\mathbb{R}^5$? If yes, why. If no, why not. We only accept simple correctly formulated answers.

5. Find orthogonal bases for the null space and for the range of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 1 & 1 & 0 & 4 \\ 2 & 0 & 3 & 8 \\ 1 & 1 & 3 & 6 \end{bmatrix}$$

Use MATLAB and it becomes easy! Look up `null` and `colspace` and `orth`. We only ask for orthogonal.

6. This problem consists of 2 steps. First find the product AB (MATLAB writes $A * B$) of the two matrices

$$A = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & c & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & x & 0 \\ 0 & 1 & y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & z & 1 \end{bmatrix}$$

Second, use the insight that the result AB gives to compute the giant product of matrices A^{7576}.

7. This problem consists of 2 steps. First find the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 1 & x & 0 & 0 & 0 \\ 0 & 1 & y & 0 & 0 \\ 0 & 0 & 1 & z & 0 \\ 0 & 0 & 0 & 1 & w \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Second, use the insight that the result A^{-1} gives to compute the inverse B^{-1} of the $n \times n$ matrix

$$B = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & -1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & -1 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & -1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$