1 What is a Group? With Exercises

In class we axiomatized a group as a structure $(G, \cdot, ^{-1}, e)$, often simply written G, where

$x \cdot y$ is a two-variable function called product,
x^{-1} is a one-variable function called inverse, and
e is a constant called unity.

Instead of $x \cdot y$ one usually writes xy. We gave as axioms

(a) $\forall x, y, z\ (x(yz) = (xy)z)$
(i) $\forall x\ (xx^{-1} = e)$
(e) $\forall x\ (xe = x)$

Using *only* these axioms, we proved in class the following Propositions for all groups:

Proposition (1): $\forall x, y, z\ (xz = yz \to x = y)$.

You can use this outcome on exams. Nevertheless, can you prove this from only the axioms above?

Proposition (2): $\forall x\ (ex = x)$.

You can use this outcome on exams. Nevertheless, can you prove this from only the axioms and Proposition (1)?

Proposition (3): $\forall x\ (x^{-1}x = e)$.

You can use this outcome on exams. Nevertheless, can you prove this from only the axioms and Propositions (1) and (2)?

A group is call *abelian*, or commutative, when it satisfies the extra property

(c) $\forall x, y\ (xy = yx)$

For abelian group one often uses notation $(G, +, -, 0)$, so $x + y$ and $-x$ and 0.