Name _____

EXAM 1

MATH 025.1001 The Nature of Mathematics

CU114 19 September 2003, 1:00 PM

1. We already know the whole numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on.

(a) A subcollection of the whole numbers are called the *prime* numbers. Examples of prime numbers are 2, 3, 5, 7, 11, and so on. Give a mathematical definition of prime number.

(b) From class we know that someone proved that all whole numbers are sums of 4 squares. Now observe that

$$1 = 1^2 + 0^2$$
$$5 = 2^2 + 1^2$$
$$9 = 3^2 + 0^2$$
$$13 = 3^2 + 2^2$$

Based on these four examples, make a conjecture about the whole numbers which are multiples of 4, plus 1. Bonus question: Try to prove the conjecture, or disprove the conjecture.

2. In class we discussed the game of nim.

(a) Suppose you are a player in the game of nim. Before you are two piles of matches, one with 34 matches, the other with 20 matches. Give a winning move.

(b) Suppose you are a player in the game of nim. Before you are four piles of matches, with 3, 4, 5, and 6 matches respectively. Give a winning move. Note: There may be more than one possible winning move; just pick one.

3. Imagine that you are a smurf. Every evening you and another smurf come together for a quiet get-together during which neither one is allowed to talk or signal to the other. It is known to both that at least one has a black dot on his cap. Once a smurf knows that (s)he has a black dot on his cap, (s)he will not come to the next get-together.

(a) Suppose at the first meeting you see no dot on the other smurf. What would you do, and why?

(b) Suppose at the first meeting you see a dot on the other smurf. What would you do, and why?

4. Suppose you are given three coins, say coins A, B, and C, and a balance. All coins have a different weight. You are supposed to find out which one is the lightest, which one is in the middle, and which one is heaviest.

(a) Give a method by which to solve the problem in 3 weighings.

(b) Do a feasibility study to determine whether or not you can solve the problem in 2 weighings.