Name _____

# EXAM 2

## MATH 025.1001 The Nature of Mathematics

## CU118 1 March 2004, 12:00 PM

1. (a) Suppose three smurfs are sitting around the table. Imagine that you are one of them. Suppose all know that at least one of the smurfs has a black dot on his cap. Whenever someone knows for sure that (s)he has a black dot, (s)he is supposed to get up and leave. You look around and see no black dot on any of the other smurf's caps. Question: Do you have a black dot on your cap, or don't you?

(b) Suppose three smurfs are sitting around the table. Imagine that you are one of them. Suppose all know that either one or two smurfs have a black dot on their cap. Whenever someone knows for sure that (s)he has a black dot, (s)he is supposed to get up and leave. You look around and see just one black dot on one other smurf's cap. Question: Do you know that you have a black dot on your cap, or don't you? Some time passes, and suddenly the smurf which you see has a black dot, says "ah, of course," and gets up and leaves. Question: Do you know that you have a black dot on your cap, or don't you?

2. (a) The standard Fibonacci sequence is defined by

$$F(0) = 0$$
$$F(1) = 1$$
$$F(n) = F(n-1) + F(n-2), \quad \text{for all } n \geq 2$$

Calculate $F(10)$, $F(11)$, and $F(12)$.

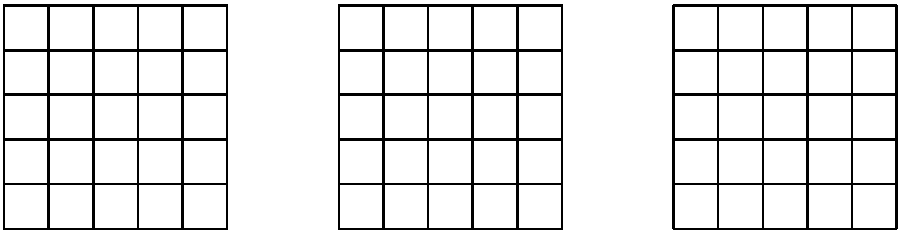
(b) The golden ratio $g$ equals

$$g = \frac{1+\sqrt{5}}{2} = 1.6180339887498948482 \text{ approximately}$$

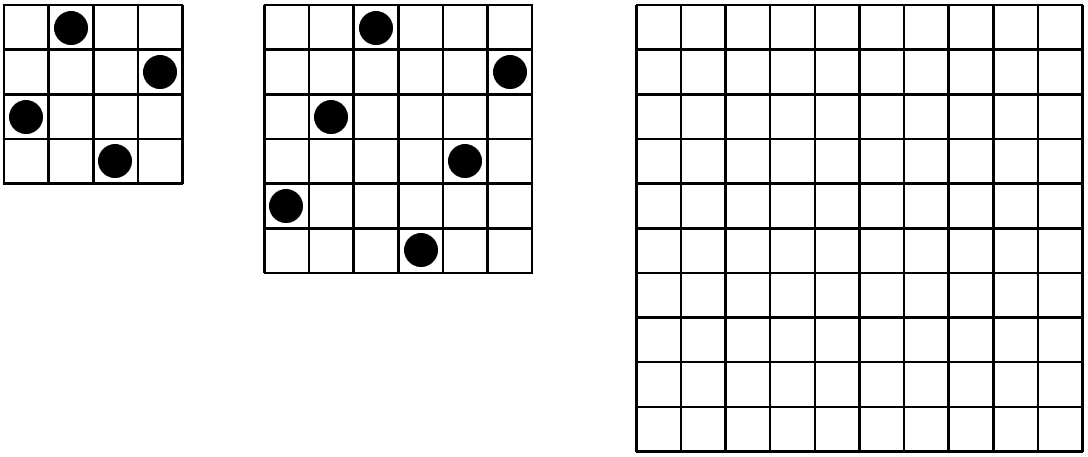
The golden ratio satisfies the nice equation $g^2 = g + 1$. Calculate $g^2$ with 19 digits precision after the decimal point.

(c) Also calculate $1/g$ with 19 digits precision after the decimal point.

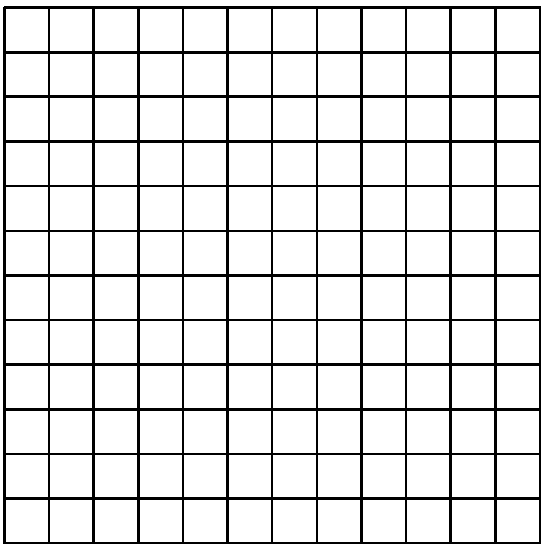
3. (a) Find a way to put 5 queens on a 5 by 5 size board such that they can not strike one another (the three boards below are for trials).



(b) Below are two examples of boards, one 4 by 4, the other 6 by 6, such that no queen can strike another. Use these examples as hints to find a way to put 10 queens on a 10 by 10 size board such that they can not strike one another.



(c) Bonus problem: Use the examples of problem (b) above to also find a solution for 12 queens on a 12 by 12 size board.



(d) Try this one last: Find a way to put 8 queens on an 8 by 8 size board such that they can not strike one another (the boards below are for trials).