- A take-home homework problem.
- Due date: Wednesday 3 December 2014.

• Purpose: Learn the enormous value of abstraction and proof. When checking the *proofs* in introductory linear algebra, many results remain valid when we generalize from the reals  $\mathbb{R}$  or complex numbers  $\mathbb{C}$  to arbitrary fields, like finite fields  $\mathbb{Z}_5$  or  $\mathbb{Z}_{17}$ , or fields like  $\mathbb{Q}(x)$ .

• Hint towards a solution of the problem: Almost-copy a proof from any decent introductory linear algebra book.

• **Problem**: Let a, b, c, d be elements of a field F. Prove that the following are equivalent:

- 1. For all  $p, q \in F$  there are unique  $x, y \in F$  so that  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$ .
- 2.  $ad bc \neq 0$ .